

# Political Economy

## Optional intermediary exam

### Solutions

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The exam lasts 120 minutes. Documents are not allowed. The use of a calculator or of any other electronic devices is not allowed.

#### Exercise 1

6 points

Let us consider an economy populated by  $N$  individuals indexed by  $i = 1, \dots, N$ . Each voter has preferences over a publicly provided good  $y$  and private consumption  $c_i$ . Voter  $i$ 's preferences are represented by the following utility function:

$$U_i = c_i + \alpha_i \log(y),$$

where  $\alpha_i$  is specific to each agent. The mean of this parameter in the population is  $\bar{\alpha}$ .

Each individual is endowed with 1 unit of the private good. The technology used to produce the public good is such that 1 unit of private good is required to produce 1 unit of public good. The government raises a per capita tax  $q$  to finance the production of the public so that  $y = Nq$ . Hence, agent  $i$ 's budget constraint is  $c_i \leq 1 - q$ , and her indirect utility function is:

$$V_i(q, \alpha_i) = 1 - q + \alpha_i \log(Nq).$$

1. Give individual  $i$ 's bliss point, i.e. her preferred policy  $q_i^*$ .

1

The policy preferred by individual  $i$  is the one that maximizes its indirect utility function. The associated first order condition can be written as:

$$\frac{\partial V_i(q, \alpha_i)}{\partial q} = 0 \Leftrightarrow -1 + \alpha_i \frac{1}{q} = 0,$$

which gives:

$$q_i^* = \alpha_i.$$

2. Assume that the social welfare function is the sum of individuals' utility functions. Show that the socially optimal policy  $q^*$  can be written as:

1

$$q^* = \frac{\sum_{i=1}^N \alpha_i}{N} = \bar{\alpha}.$$

Let us write the social welfare function as:

$$\mathbb{W} = \sum_{i=1}^N V_i(q, \alpha_i) = N - Nq + \log(Nq) \sum_{i=1}^N \alpha_i.$$

The optimality condition is:

$$\frac{\partial \mathbb{W}}{\partial q} = 0 \Leftrightarrow -N + \frac{1}{q} \sum_{i=1}^N \alpha_i,$$

which gives:

$$q^* = \frac{\sum_{i=1}^N \alpha_i}{N} = \bar{\alpha}.$$

Let us model Downsian political competition. Two political parties  $P = A, B$  compete for office. They are only office-motivated. They announce platforms  $q_A$  and  $q_B$  to which they can commit. The election takes place under the majority rule. Each voter  $i$  votes for the party that will provide him with the highest utility. Let us note  $q_m^*$  the median voter's bliss point.  $\pi_P$  is the vote share of party  $P$ . The probability of  $P$  winning the election is  $\mathbb{P}(\pi_P \geq \frac{1}{2})$ .

- Carefully describe the political competition and its outcome. That is, determine parties probabilities of winning, their equilibrium platforms and the one that is finally implemented.

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Since political parties are only office-motivated, they only seek to maximize their probability to be elected. The only tool they can use to achieve this objective is the platform they announce. Each party's optimization program can be written as:

$$\max_{q_P} \mathbb{P} \left( \pi_P \geq \frac{1}{2} \right),$$

where:

$$\mathbb{P} \left( \pi_P \geq \frac{1}{2} \right) = \begin{cases} 1, & \text{if } \pi_P > \frac{1}{2}, \\ \frac{1}{2}, & \text{if } \pi_P = \frac{1}{2}, \\ 0, & \text{if } \pi_P < \frac{1}{2}. \end{cases}$$

Since individuals have single peaked preferences ( $V_i$  is increasing if  $q < q_i^*$  and decreasing if  $q > q_i^*$ ), they will vote for the party whose platform is the closest from their bliss point. According to the median voter theorem, the winning policy platform is the one preferred by the median voter, i.e.  $q_m^*$ . This platform will be the one announced by both parties, i.e. political competition will result in the following equilibrium outcome:

$$q_A = q_B = q_m^*.$$

To prove this, consider a situation in which party  $A$  does not announce  $q_m^*$ . Then, party  $B$ 's best response is to announce a platform that is closer from  $q_m^*$  than  $q_A$ . Party  $B$  will then win the election for sure. So, party  $A$ 's best response is to announce some platform that is closer from  $q_m^*$  than  $q_B$ . Party

A would then win the election for sure. And so on, and so forth. Such that the above described equilibrium is unique.

As both parties offer the same platform and as parties have no incentives not to implement the platform they announced, each party has the same probability to get elected and the finally implemented policy is the one preferred by the median voter.

4. Under what conditions will Downsian political competition achieve the social optimum? Comment. 1

We showed that the social optimum is achieved if the implemented policy is such that  $q = \bar{\alpha}$ . In contrast, political competition results in implementing  $q = q_m^* \equiv \alpha_m$ , where  $\alpha_m$  is the median value of  $\alpha$  within the society.

So, Downsian political competition will only achieve the social optimum if the distribution  $\alpha$  is such that the mean equals the median.

There is a priori no reason that this will be the case. Accordingly, political competition does not warrant that the social optimum will be implemented.

## Exercise 2

10 points

Consider a two-period model with politicians that can be *congruent* or *dissonant*. The share of congruent politicians in the pool of potential leaders is  $\pi$ . In each period  $t = 1, 2$ , the leader in charge chooses a state-dependent policy  $e_t(s_t, i)$  where  $i \in \{C, D\}$  is the type of the incumbent politician and  $s_t \in \{0, 1\}$  is the state of the world at time  $t$ . Each state can occur with equal probability and is only observed by the incumbent politician. Citizens, which are represented by a single representative voter, receive  $V_t = \Delta$  if  $e_t = s_t$ , and  $V_t = 0$  otherwise. Citizens do not observe politicians' type. Both citizens and politicians discount the future at rate  $\beta \in [0, 1]$ . Congruent politicians choose  $e_t$  to maximize citizens' payoff. In contrast, dissonant politicians receive a private rent  $r_t$  for setting  $e_t \neq s_t$ . Rent are drawn from the cumulative distribution function  $G(r)$  with mean  $\mu$  and finite support  $[0, R]$ . In each period, the incumbent politician receives wage  $E$  for being in office. We assume  $R > \beta(\mu + E)$ .

The timing and the election rules of this model are as follow. (i) A random incumbent is selected from the pool of potential leaders and  $r_1$  is drawn from  $G(r)$  if she is dissonant. (ii) Nature determines the state of the world  $s_1$ . (iii) The incumbent politician chooses  $e_1$  and receives her payoff. (iv) Voters observe  $V_1$  and decide whether to reelect the incumbent or to replace her by a challenger drawn from the pool of potential leaders. (v)  $r_2$  is drawn from  $G(r)$  if the incumbent politician is dissonant, nature determines  $s_2$ , the incumbent politician chooses  $e_2$ , etc. The world ends at the end of period 2.

1. Let us note  $\lambda$  the probability that a dissonant incumbent behave congruently. Show that voters will always reelect an incumbent that chooses  $e_1 = s_1$ . 1

The questions voters ask themselves at the end of period 1 is whether or not they should reelect the incumbent. They need to take this decision, knowing that they will be better off in period 2 with a congruent leader. Their decisions is based on what they observe in period 1, i.e. their payoff that can be either  $\Delta$  or 0. Using Bayes' rule, the probability that the incumbent

is congruent conditional on receiving  $V_1 = \Delta$  can be written as:

$$P(i = C|V_1 = \Delta) = \frac{P(i = C) \times P(V_1 = \Delta|i = C)}{P(V_1 = \Delta|i = C) \times P(i = C) + P(V_1 = \Delta|i = D) \times P(i = D)},$$

which simplifies as:

$$P(i = C|V_1 = \Delta) = \frac{\pi \times 1}{\pi + (1 - \pi)\lambda},$$

where  $\lambda = P(V_1 = \Delta|i = D)$ , i.e. the probability that a dissonant incumbent behave congruently.

Since  $\forall \lambda : P(i = C|V_1 = \Delta) > \pi$ , voters will always reelect an incumbent that delivers  $\Delta$  in period 1. In contrast, voters would never reelect an incumbent that delivers 0 in period 1.

2. What are politicians' optimal decisions in period 2? 1

As time is over at the end of period 2, a dissonant politician will always set  $e_2 \neq s_2$  and choose to extract rent. In contrast, a congruent politician will set  $e_2 = s_2$ .

3. What are politicians' optimal decisions in period 1? Give the analytical value of  $\lambda$  and explain how it varies with relevant parameters. 2

In period 1, a congruent politician will simply set  $e_1 = s_1$  because she has the same objective as voters. In contrast, a dissonant politician faces a trade-off. On the one hand, she might behave congruently, i.e. set  $e_1 = s_1$ , and be sure to be reelected (see above) such that she will be able to extract rent in period 2. On the other hand, she might behave dissonantly, i.e. set  $e_2 \neq s_2$ , and extract rent in period 2 while being sure not to be reelected. A dissonant incumbent will behave congruently in period 1 if and only if:

$$E + \beta(E + \mathbb{E}(r_2)) > E + r_1 + \beta \times 0,$$

which can be rewritten as:

$$r_1 < \beta(\mu + E).$$

Since  $G(r)$  is the cumulative distribution function of rents, we get:

$$\lambda = G(\beta(\mu + E)).$$

$\lambda$  is increasing with  $\beta$ ,  $\mu$ , and  $E$ . It is increasing with  $\beta$  because the less politicians discount the future, the more incentives they have to stay in office (payoff is implicitly set to 0 in period 2 if the politician is not in office). It is increasing in  $E$  and  $\mu$  because these are the two components of period 2's payoff for a dissonant politicians. Hence, increasing  $E$  would reduce incentives for dissonant politicians to behave dissonantly in period 1.

4. Write down  $\mathbb{V}_1$  and  $\mathbb{V}_2$ , the ex-ante voters' welfare in period 1 and 2. Discuss how these quantities and total ex-ante welfare  $\mathbb{W}$  vary with  $\pi$  and  $\lambda$ . Interpret. 2

$\mathbb{V}_1$  can be written as:

$$\mathbb{V}_1 = \pi\Delta + (1 - \pi)\lambda\Delta.$$

$\mathbb{V}_2$  can be written as:

$$\mathbb{V}_2 = \pi\Delta + (1 - \pi)(1 - \lambda)\pi\Delta.$$

$\mathbb{V}_1$  and  $\mathbb{V}_2$  are both increasing with  $\pi$  as an increase in  $\pi$  simply means that the chance to pick a congruent politician is higher. However,  $\mathbb{V}_1$  is increasing with  $\Delta$  while  $\mathbb{V}_2$  is decreasing with  $\Delta$  because  $\lambda$  improves dissonant politicians' behavior in period 1 while making it more difficult to detect them and to get rid of them for period 2.

Writing total welfare as:

$$\mathbb{W} = \mathbb{V}_1 + \beta\mathbb{V}_2,$$

it comes out that  $\lambda$  has a total positive effect on  $\mathbb{W}$  as the positive effect dominates the negative one.

5. Write down  $\mathbb{R}_1$  and  $\mathbb{R}_2$ , the expected values of rents in period 1 and 2. Which one is larger? Discuss how these quantities vary with  $\pi$  and  $\lambda$ . Interpret. 2  
*Hint: Let us note  $\mu'$  the mean of  $r_t$  over  $[\beta(\mu + E), R]$  and assume that  $\mu' = \mu + \varepsilon$ , with  $\varepsilon \approx 0$ .*

In period 2, rents will be extracted if and only if a dissonant politician was first picked and then reelected. This occurs with probability  $(1 - \pi)\lambda$ . Hence, the expected value of rents in period 2 is:

$$\mathbb{R}_2 = (1 - \pi)\lambda\mathbb{E}(r_2) = (1 - \pi)\lambda\mu.$$

In period 1, rents will be extracted if and only if a dissonant politician is picked and choose not to behave congruently but to extract rents. Hence, the expected value of rents in period 1 is:

$$\mathbb{R}_1 = (1 - \pi)(1 - \lambda)\mathbb{E}(r_1 | r_1 > \beta(\mu + E)) = (1 - \pi)(1 - \lambda)\mu'.$$

It is clear that both  $\mathbb{R}_1$  and  $\mathbb{R}_2$  are decreasing with  $\pi$  as this parameter represent the probability to face a congruent politicians.

In contrast,  $\mathbb{R}_1$  is decreasing with  $\lambda$  while  $\mathbb{R}_2$  is increasing with  $\lambda$ . This simply reflect the fact that a high  $\lambda$  will push dissonant politicians not to extract rents in period 1 and favor their survival up to period 2 (and the associated rent extraction).

Furthermore, note that, assuming that  $\mu' \approx \mu$ , the expected value of extracted rents is higher in period 1 if  $\lambda$  is smaller than 1/2.

6. Assume the representative voter can set the incumbent's wage  $E$  at cost  $C(E)$ . Write down the optimization program that would allow to optimally choose  $E$ . Explain the trade-off faced by the representative voter. 2

The optimization program of the representative voter is:

$$\max_E \mathbb{W} - C(E),$$

where  $\mathbb{W}$  depends on  $\lambda$  which is itself a function of  $E$ .

The representative voter faces a trade-off when setting  $E$  as she must pay for it while knowing that increasing  $E$  would increase  $\lambda$  and thus  $\mathbb{W}$ . At the optimum,  $E$  will be set such that the marginal cost of politicians' wage equals the marginal benefits of  $E$ , i.e. the disciplining power of incentives. Formally, the optimality condition is:

$$\frac{\partial \mathbb{W}}{\partial E} = \frac{\partial C(E)}{\partial E}.$$

### Question

4 points

Discuss the role of information in the relationship between politicians and voters.

It is worth considering the role of information in the relationship between politicians and voters via a principal-agent framework. In such a framework, voters are principals and elected politicians are agents.

Two important information issues arise in such a framework. First, voters imperfectly observe politicians' characteristics, making difficult to choose good politicians as leaders. This is an adverse selection problem. Second, voters imperfectly observe politician's actions once the latter are in office, making it difficult to monitor incumbents. This a a moral hazard problem.

Elections can play a role in alleviating the moral hazard issue by letting voters choose to reelect or not an incumbent politicians depending on the actions they observed. This however strongly depends on the quality of the information they have access to. It is also worth noting that a better quality information on leaders' actions might help detect a bad politician but will also reduce the incentives for bad politicians to correctly as they know they might be detected and not reelected.

If politicians are elected, better information on candidates can help to select better politicians in two ways. First, by simply making observable prospective leader's characteristics. Second, by the disciplining effect of reelection that might be anticipated by candidates who might self-select knowing that voters will have information on their future actions if they get elected.