Public Economics

Problem set 3

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Exercise 1

Consider a representative consumer that has the following utility function:

$$U(x_1, x_2, l) = \frac{1}{\alpha}x_1^\alpha + \frac{1}{\beta}x_2^\beta - l,$$

where $(\alpha, \beta) \in (0,1)^2$, x_i denotes consumption of good i and l is labor time. Hourly wage is $w = 1$, such that the consumer's budget constraint is simply:

$$x_1 q_1 + x_2 q_2 \leq l,$$

where q_i is the unit price of good i payed by the consumer.

1. Determine the consumer's demand in both goods. Calculate ε_1 and ε_2, the price-elasticities of goods.

For some (good) reason, the government want to set taxes on consumption goods. She chooses to set a unit tax t_i on good i, such that $q_i = p_i + t_i$. From now on, assume that $\alpha = \frac{1}{2}$ and $\beta = \frac{2}{3}$. Also assume that producers' unit revenues are such that $p_1 = p_2 = 1$.

2. Use the inverse elasticity rule to show that optimal taxes will be such that:

$$t_2 = 2\frac{1}{1 + \frac{3}{t_1}}.$$

3. Which unit-tax is the largest? Why was it to be expected?

4. Determine t_1 and t_2 such that the government has a total revenue of $R = 0.375$. *Note:* $x = 0.6$ is the solution the following equation:

$$0.375 \approx x(1+x)^{-2} + \left(2\frac{1}{1+\frac{3}{x}}\right)\left(1 + 2\frac{1}{1+\frac{3}{x}}\right)^{-3}.$$

Exercise 2

Consider an individual that is isolated within the society. Her utility function is:

$$U(c, l) = c - \frac{1}{1+\mu} l^{1+\mu},$$

where $\mu > 0$, c denotes consumption, and l is labor time. Hourly wage is $w = 1$, such that the budget constraint is simply:

$$c \leq l(1-t),$$

where t is the tax rate faced by the individual.

1. Determine l^*, the individual's optimal labor supply given a tax rate t.
2. Determine ε, the individual's labor supply elasticity with respect to net of tax wage.

Government's revenue from taxing the individual is $R = l^* t$. Let us assume that the social welfare function used by the government is:

$$\mathbb{W} = \alpha R + U(l^*),$$

where $\alpha > 1$.

3. Interpret α.
4. Show that the government will choose:

$$t^* = \frac{(\alpha - 1)\mu}{(\alpha - 1)(1+\mu) + 1}.$$

5. Comment on how t^* varies with α and μ.