

# Public Economics

## Problem set 2

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### Exercise 1

Let us consider an economy populated by 2 consumers— $A$  and  $B$ —who are endowed with 1 unit of income and derive utility from the consumption of a private good  $x$  and a pure public good  $G$ . Individual  $i$  utility function is given by:

$$U^i = \log(x_i) + \log(G),$$

where  $x_i = 1 - g_i$  denotes consumption of the private good by consumer  $i$ , and  $G = g_A + g_B$  is the total quantity public good that is produced from individuals contributions.

1. Determine individual  $A$ 's private provision of the public good when considering  $g_B$  as given.
2. Determine individual  $B$ 's private provision of the public good when considering  $g_A$  as given.
3. Use the two reaction functions to find  $G^*$ , the quantity of public good that is supplied at the Nash equilibrium.
4. Determine  $\bar{G}$ , the efficient level of public good provision. Contrast it with the decentralized equilibrium.
5. Show that producing  $\bar{G}$  is Pareto-superior to producing  $G^*$ .
6. Show that private contribution required to produce  $\bar{G}$  cannot be sustained without the intervention of some third party that would be able to constrain individuals' contributions.

### Exercise 2

Let us consider an economy populated by 2 individuals— $A$  and  $B$ —who consume 2 goods—1 and 2. Individuals' utility function are:

$$U^A = \log(x_1^A) + x_2^A - \frac{1}{2} \log(x_1^B),$$

and,

$$U^B = \log(x_1^B) + x_2^B - \frac{1}{2} \log(x_1^A),$$

where  $x_j^i$  is the quantity of good  $j$  consumed by individual  $i$ . Each individual is endowed with 1 unit of income. Let the unit prices of both goods be 1.

1. Calculate the decentralized equilibrium situation of this economy.
2. Calculate the social optimum if the social welfare function is the sum of individuals' utility functions.
3. Check that the social optimum is Pareto-superior to the decentralized one.
4. Show that the social optimum can be reached in a decentralized framework thanks to a tax  $t$  placed on good 1 (so, the price of this good is now  $1 + t$ ), with the tax revenues returned equally to consumers via a lump-sum transfer  $T$ .