

# Public Economics

## Optional intermediary exam

Marc Sangnier - marc.sangnier@univ-amu.fr

February 13<sup>th</sup>, 2015

The exam lasts 90 minutes. Documents are not allowed. The use of a calculator or of any other electronic devices is not allowed. You can answer either in French or in English.

### Exercise 1

8 points

In a *transferable voting system* each voter provides a ranking of options. If no option achieves the majority, the option with the lowest number of first-choice votes is eliminated and the votes that were attached to it are transferred to the second-choice options (for voters who first-choice was eliminated). This process proceeds until an option achieves a majority.

1. Define what is a Condorcet winner. 1
2. Is it possible for an option that is no one's first choice to win under a transferable voting system? 2

Consider the following preferences of five voters  $i = 1, \dots, 5$  over three alternatives  $a$ ,  $b$ , and  $c$ :

	1	2	3	4	5
Most preferred alternative	a	b	b	c	c
	b	a	a	a	a
Least preferred alternative	c	c	c	b	b

3. Assume that voters truly express their preferences. What will be the selected option under a transferable voting system? Is this the Condorcet winner? 2
4. Show how strategic voting can affect the outcome of the vote. What will be the outcome of the vote if voters vote strategically? 3

**Exercise 2**

6 points

Let us consider an economy populated by 2 individuals— $A$  and  $B$ —who consume 2 goods—1 and 2. Individuals' utility function are:

$$U^A = \log(x_1^A) + \log(x_2^A) + \frac{1}{2} \log(x_1^B),$$

and

$$U^B = \log(x_1^B) + \log(x_2^B) + \frac{1}{2} \log(x_1^A),$$

where  $x_j^i$  is the quantity of good  $j$  consumed by individual  $i$ . Each individual is endowed with 1 unit of income. Let the unit prices of both goods be 1.

1. Calculate the decentralized equilibrium situation of this economy. 1
2. Calculate the social optimum if the social welfare function is the sum of individuals' utility functions. 1
3. Compare quantities of good 1 under both situations. Comment. 2
4. Show that the social optimum can be reached in a decentralized framework thanks to a subsidy  $s$  placed on good 1 (so, the price of this good is now  $1 - s$ ), with the cost of this subsidy covered by a lump-sum tax  $T$  on each consumer. 2

**Exercise 3**

6 points

This exercise describes what is known as the *tragedy of the commons*. Consider a lake that can be freely accessed by a potentially infinite number of fishermen. The cost of sending a boat out on the lake is  $r > 0$ . When  $b$  boats are sent out onto the lake,  $f(b) = \sqrt{b}$  fishes are caught in total. So, each boat catches  $f(b)/b$  fishes. The unit price at which fishermen can sell fishes is  $p > 0$ , it is not affected by the level of the catch from the lake (i.e. we are reasoning in partial equilibrium). Fishermen's outside option is 0 if they do not fish.

1. Show that the equilibrium number of boats sent out on the lake if fishermen take decentralized decisions can be expressed a: 1

$$b^* = \left(\frac{p}{r}\right)^2$$

2. Determine  $b^o$ , the number of boats that maximizes total social surplus. 1
3. Compare  $b^o$  and  $b^*$ . Why don't the two values coincide? 2
4. What per-boat tax  $t$  would allow to restore efficiency? 2