Public Economics

Lecture 3: Social choice and social welfare

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Basic question

- Let X be the set of mutually exclusive social states (complete descriptions of all relevant aspects of a society).
- Let N be the set of individuals living in the society. Individuals are indexed by $i \in \{1, \ldots, n\}$.

Examples:

- $X = \mathbb{R}^n_+$, the set of *all* income distributions.
- $X = \mathbb{R}^{n \times m}_+$, the set of *all* allocations of m goods between the n individuals.

- Let $\succsim$ be a “normal” relation of preference (reflexive, complete, and transitive).
- $x \succsim_i y$ means that individual i weakly prefers situation x over situation y.
- $x \succ_i y$ means that individual i strictly prefers situation x over situation y.
- $x \sim_i y$ means that individual i is indifferent between situations x and y.

Arrow (1950): How can we compare the various elements of X on the basis their "social goodness"? How construct an aggregate relation of preference?

- Dictatorship of individual h:
  $x \succsim y \Leftrightarrow x \succsim_h y$.
- Exogenous code:
  $x \succsim y$ even if $y \succ_i x, \forall i \in N$.

Can we find a "satisfying" collective decision rule?

Unanimity rule

Unanimity rule:
$x \succsim y \Leftrightarrow x \succsim_i y, \forall i \in N.$

- Pareto criterion;
- Nice, but incomplete: alternatives for which individuals' preferences conflict cannot be ranked.

Majority rule

Majority rule:
$x \succsim y \Leftrightarrow x\#\{i \in N : x \succsim_i y\} \geq \#\{i \in N : y \succsim_i x\}.$

- Widely used;
- Does not always lead to a transitive ranking of alternative situations (Condorcet paradox).

Condorcet winner

Principle of majority voting for more than two options:
Vote over two alternatives at a time.

The option that defeats all others in pairwise majority voting is called a Condorcet winner.

Condorcet paradox

Three individuals, three choices.

Individual 1	Individual 2	Individual 3
Marine	Nicolas	François
Nicolas	François	Marine
François	Marine	Nicolas

A majority (1 and 3) prefers M. to N. $\Rightarrow$ Marine $\succ$ Nicolas.
A majority (1 and 2) prefers N. to F. $\Rightarrow$ Nicolas $\succ$ François.
Transitivity of the $\succ$ relation would imply that Marine $\succ$ François.
A majority (2 and 3) prefers F. to M. $\Rightarrow$ François $\succ$ Marine.
Transitivity is violated.

Borda rule

- Idea: Each individual assigns a score to each alternative situation. Then, situations are ranked on the basis of the sum of scores over all individuals.
- The "Borda score" $\mathbb{B}$ of situation x assigned by individual i is the number of situations that individual i considers weakly worse than x:
  $\mathbb{B}_i(x) = \# \{y \in X : x \succsim y\}$.
  The total "Borda schore" of situation x is:
  $\mathbb{B}(x) = \sum_{i=1}^{n} \mathbb{B}_i(x)$.
- $x \succ y \Leftrightarrow \mathbb{B}(x) > \mathbb{B}(y)$ and $x \sim y \Leftrightarrow \mathbb{B}(x) = \mathbb{B}(y)$.
- This decision's rule works only if X is finite.

Illustration

Three individuals, four choices.

Individual 1		Individual 2		Individual 3	
Marine	4	Nicolas	4	François	4
Nicolas	3	François	3	Marine	3
Jean-Luc	2	Jean-Luc	2	Nicolas	2
François	1	Marine	1	Jean-Luc	1

$\mathbb{B}(\text{Marine}) = 8$, $\mathbb{B}(\text{Nicolas}) = 9$, $\mathbb{B}(\text{François}) = 8$, $\mathbb{B}(\text{Jean-Luc}) = 5$

Thus:

$\text{Nicolas} \succ \text{Marine} \sim \text{Francois} \succ \text{Jean-Luc}.$

Jean-Luc seems irrelevant, but...
if two individuals slightly change Jean-Luc's ranking.

Individual 1		Individual 2		Individual 3	
Marine	4	Nicolas	4	François	4
Jean-Luc $\uparrow$	3	François	3	Marine	3
Nicolas $\downarrow$	2	Marine $\uparrow$	2	Nicolas	2
François	1	Jean-Luc $\downarrow$	1	Jean-Luc	1

$\mathbb{B}(\text{Marine}) = 9$, $\mathbb{B}(\text{Nicolas}) = 8$, $\mathbb{B}(\text{François}) = 8$,
$\mathbb{B}(\text{Jean-Luc}) = 5$
Thus:
Marine $\succ$ Nicolas $\sim$ Francois $\succ$ Jean-Luc.
Social ranking of Marine and Nicolas depends upon the individual rankings of Jean-Luc against Nicolas against Jean-Luc or Marine.

Jean-Luc seems irrelevant, but...
if Jean-Luc steps out.

Individual 1		Individual 2		Individual 3	
Marine	3	Nicolas	3	François	3
Nicolas	2	François	2	Marine	2
François	1	Marine	1	Nicolas	1

$\mathbb{B}(\text{Marine}) = 6$, $\mathbb{B}(\text{Nicolas}) = 6$, $\mathbb{B}(\text{François}) = 6$

Thus:

Marine $\sim$ Nicolas $\sim$ Francois.

Here, again, social ranking is not stable.

Can we find better decision rules?

- Arrow (1951) proposes five axioms that should be satisfied by any collective decision rule.
- He shows that there is no rule that satisfies all axioms (impossibility theorem).
- Pessimism on the prospect of obtaining a good definition of general interest as a function of the individual interest.

Axioms

1. Non-dictatorship:

$$\nexists h \in N : \forall (x,y) \in X^2, x \succ_h y \Rightarrow x \succ y.$$

2. Collective rationality:
The social ranking must be a complete, transitive (and reflexive) ordering.

3. Unrestricted domain:
The decision rule must apply to all logically conceivable preferences.

4. Weak Pareto principle:

$$\forall (x, y) \in X^2 : \qquad x \succ_i y, \qquad \forall i \in N \Rightarrow x \succ y.$$

5. Binary independence for irrelevant alternatives:
   The social ranking of x and y must only depend upon the individual rankings of x and y.

Arrow's impossibility theorem

There does not exist any collective decision function that satisfies axioms 1 *to* 5.

Illustration

	Non-dictatorship	Rationality	Domain	Pareto	Binary ind.
Dictatorship		✓	✓	✓	✓
Exogenous code	✓	✓	✓		✓
Majority rule	✓		✓	✓	✓
Unanimity rule	✓		✓	✓	✓
Borda rule	✓	✓	✓	✓	

Escape out of Arrow's theorem

- Natural strategy: relaxing axioms.
- Difficult to relax non-dictatorship.
- We may relax collective rationality, in particular "completeness".
- We may relax the condition on unrestricted domain.
- We may relax the binary independence of irrelevant alternatives.
- Should we relax the weak Pareto principle?

Relax Pareto principle?

Most economists (who use the Pareto principle as the main criterion for efficiency) would say no.
Recall of Pareto principle:

- Given a set of situation $A \subset X$, a is efficient if there are no other state in A that everybody weakly prefers to a and at least somebody strictly prefers to a.

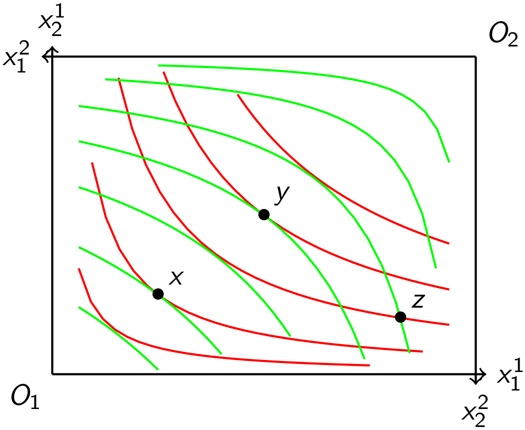
Frequent abuses of the Pareto principle:

- If $a \in A$ is efficient and $b \in A$ is not efficient, then a is socially better than b.
- Situation a is socially better than b if it is possible to compensate the losers in the move from b to a while keeping the gainers gainers.

Only one use is admissible:

- If everybody believes that x is weakly better than y, then x is socially weakly better than y.

Illustration



x and y are efficient. z is not.

$y \succ z$? Yes. $x \succ z$? No.

Sen liberal paradox

Sen (1970):

- When combined with unrestricted domain, the Pareto principle may hurt widely accepted liberal values.
- Minimal liberalism is the respect for an individual personal sphere (Mills).
- Example:
  x is a social state in which Mary sleeps on her belly and y is a social state that is identical to x in every respect other than the fact that, in y, Mary sleeps on her back. Minimal liberalism would impose, it seems, that Mary be decisive on the ranking of x and y.

- Minimal liberalism:
  There exists two individuals h and $i \in N$, and four social states w, x, y, and z. Individual h is decisive over x and y, and i is decisive over w and z.
- Sen impossibility theorem:
  There exist no collective decision function that satisfies unrestricted domain, weak Pareto principle and minimal liberalism.

Proof (example)

- A novel: *Fifty Shades of Grey* (*Lady Chatterley's Lover* in Sen's original proof).
- Two individuals: Christine is prude and Dominique is libertine.
- Four social states:
  - w, everybody reads the book;
  - x, nobody reads the book;
  - y, only Christine reads the book;
  - z, only Dominique reads the book.
- Under minimal liberalism:
  - Christine is decisive to discriminate between x and y, and between w and z;
  - Dominique is decisive to discriminate between x and z, and between w and y.

- Assume that (unrestricted domain):
  - Christine: $x \succ y \succ z \sim w$;
  - Dominique: $w \sim y \succ z \succ x$.
- Minimal liberalism: $x \succ y$ according to Christine decisiveness.
- Pareto principle: $y \succ z$ as both agree on it.
- It follows by transitivity that $x \succ z$, what violates Dominique decisiveness of Dominique who would imply $z \succ x$.

- Shows a problem between liberalism and respect of preferences when the domain is unrestricted.
- When people are allowed to have any preference (even for things that are "not of their business"), it is impossible to respect these preferences (in the Pareto sense) and the individual's sovereignty over their personal sphere.
- Sen Liberal paradox: attacks the combination of the Pareto principle and unrestricted domain.
- Suggests that unrestricted domain may be a (too) strong assumption.

Relaxing unrestricted domain

- Assume X is the set of all allocations of m goods between the n individuals, i.e. $X = \mathbb{R}_+^{n \times m}$.
- In such a context, individuals could be selfish, i.e. they care only about what they get.
- Assume also that individual have convex, continuous, and monotonic preferences.
- Still... that's not enough to escape Arrow's impossibility theorem.

Single peaked preferences

Formal definition:

> *Relation $\succsim$ is single peaked with respect to the linear order $\geq$ on X is there is $x \in X$ such that $\succsim$ is increasing with respect to $\geq$ on $\{y \in X : x \geq y\}$ and decreasing with respect to $\geq$ on $\{y \in X : y \geq x\}$.*

That is:

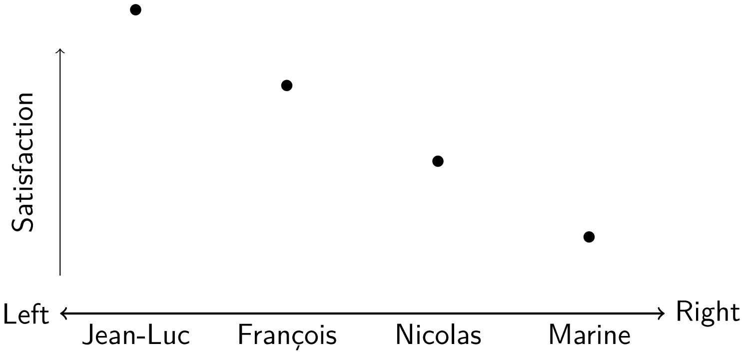
$$\text{If } x \geq z > y, \text{ then } z \succ y,$$
$$\text{if } y > z \geq x, \text{ then } z \succ y,$$

Definition with words:

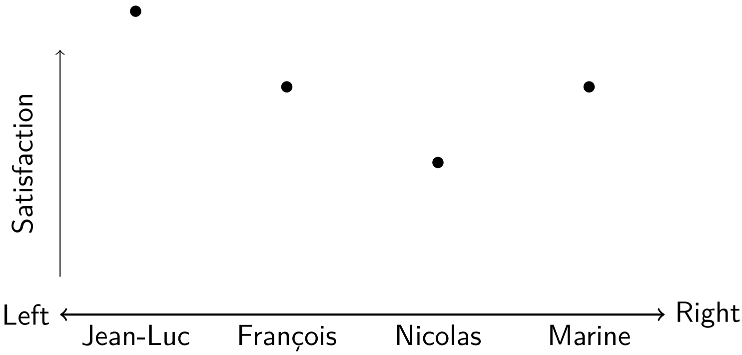
> *There is an alternative x that represents a peak of satisfaction and, moreover, satisfaction increases as we approach this peak.*

Thus, there cannot be any other peak of satisfaction. Preferences are *single peaked*.

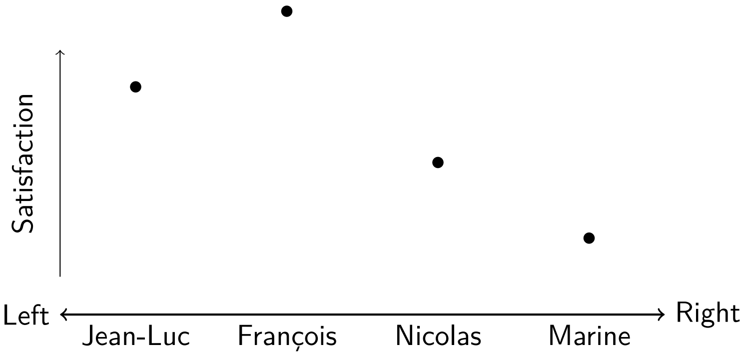
Examples



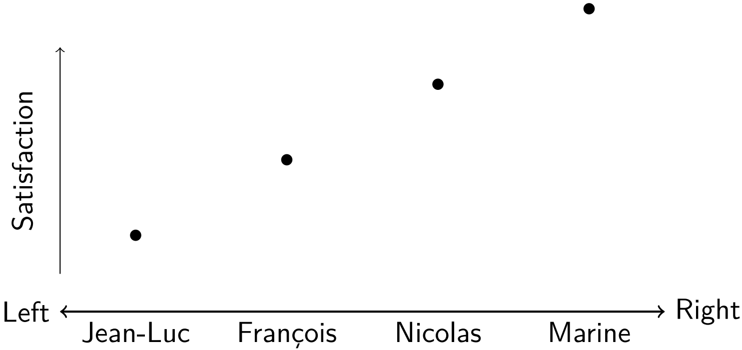
These preferences are single peaked.



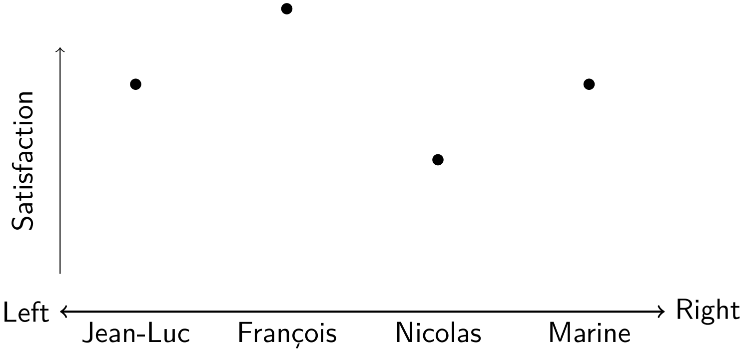
These preferences are not single peaked.



These preferences are single peaked.



These preferences are single peaked.



These preferences are not single peaked.

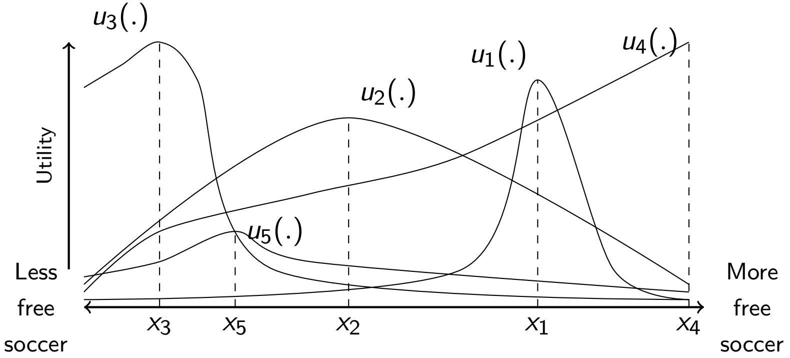
Median voter theorem

Black (1947):

> *If there is an odd number of voters, if the policy space is one-dimensional, and if the voters have single peaked preferences, then the median of the distribution of voters' preferred options is a Condorcet winner.*

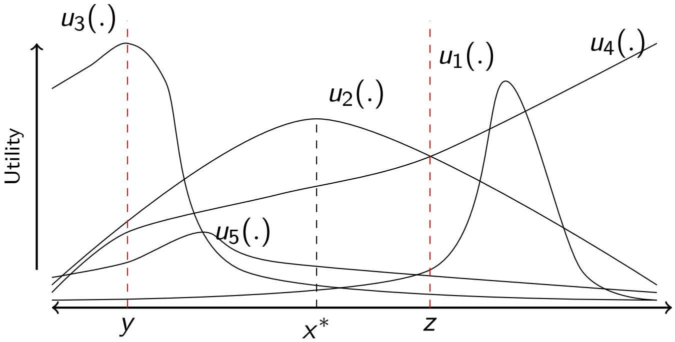
Majority rule allows to reach this outcome.

Illustration



Agent 2 is the median voter.

Proof



$x^* \succ_1 y$, $x^* \succ_2 y$, $y \succ_3 x^*$, $x^* \succ_4 y$, $x^* \sim_5 y \Rightarrow x^* \succ y$.
$z \succ_1 x^*$, $x^* \succ_2 z$, $x^* \succ_3 z$, $z \succ_4 x^*$, $x^* \succ_5 z \Rightarrow x^* \succ z$.
x^* is the Condorcet winner.

Limitations

- Does not old for multidimensional voting.
- Important restriction: number of voters must be odd.

Individual 1	Individual 2	Individual 3	Individual 4
Jean-Luc	François	Nicolas	Nicolas
François	Jean-Luc	François	François
Nicolas	Nicolas	Jean-Luc	Jean-Luc

If preferences are single peaked on the left-right axis, then:

$$\left.\begin{array}{l}\text{Jean-Luc} \sim \text{Nicolas} \\ \text{François} \succ \text{Jean-Luc}\end{array}\right\} \Rightarrow \text{François} \succ \text{Nicolas}$$

But: $\text{François} \sim \text{Nicolas},$

which is not consistent.

How to guarantee transitivity of majority voting?

- Extremal restriction condition:
  A profile of preferences satisfies the extremal restriction condition if and only if $\forall(x, y, z) \in X^3$, the existence of an individual i for which $x \succ_i y \succ_i z$ implies that $z \succ_h y \succ_h x$ for all individuals h for which $z \succ_h x$.
- Theorem by Sen and Pattanaik (1969):

  A profile of preferences satisfies the extremal restriction condition $\Leftrightarrow$ the majority rule defined on this profile is transitive.

Prediction of the median voter theorem

- The policy that will attract more votes is the one preferred by the median voter.
- Standard political competition model:
  - Politicians compete for election, i.e. they choose their platform in order to win the election.
  - The likelihood to win is higher the closer from the median voter preferred policy their platform is.
  - Platforms of the different candidates will converge (toward the policy preferred by the median voter).

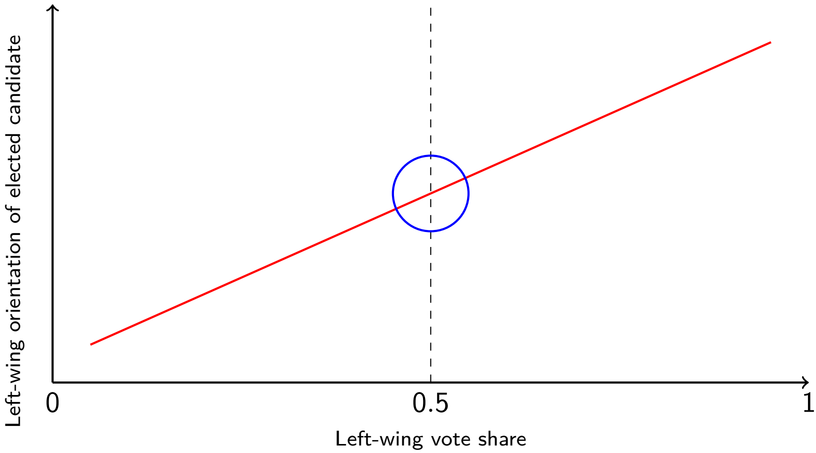
Empirical illustration

- If the population is left-wing, the winning candidate should be left-wing oriented.
- The more votes she received during the election, the more left- or -right-wing oriented is the population. Accordingly, “better” elected representatives should have more “extreme” view.
- On the opposite, candidates elected during close races should have a political orientation very close to the one of their defeated opponent.

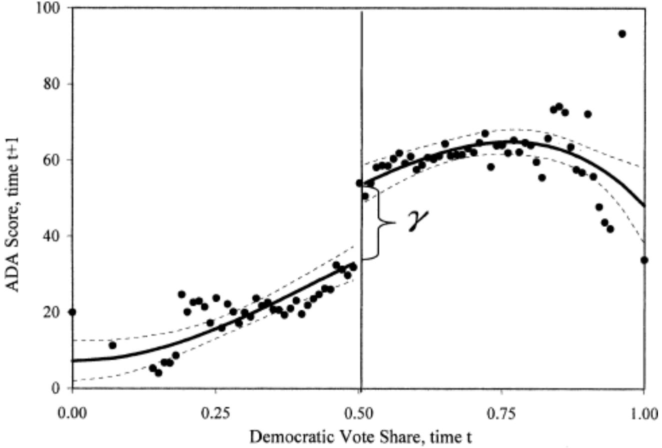
David S. Lee & Enrico Moretti & Matthew J. Butler, 2004. "Do Voters Affect Or Elect Policies? Evidence from the U. S. House," The Quarterly Journal of Economics, MIT Press, vol. 119(3), pages 807-859, August.

- Two-party context:
  Election of US House of Representatives (local election).
- Each candidate's orientation is measured using its votes in the US House.
- Identification strategy: their should be no (large) differences in political orientations of left- and right-wing candidates elected in close races.

What should we observe?



What do we observe?



Source: Lee, Moretti and Butler (2004)

Comments

- Strong assumptions about politicians' objectives, commitment and credibility.
- Votes *after* the election may not reflect electoral platforms.
- Conclusions mitigated by other papers.

More voting rules

- As majority voting may fail to select the Condorcet winner, voters may anticipate it and choose to vote for their second preferred choice to avoid the victory of a worse option.
- Such strategic voting may lead to misrepresentation (or "mis-expression") of preferences.
- Sequential selection of Condorcet winner would require multiple votes.

Approval voting

- Each voter can "approve" as many options as she wants, and the alternative with highest number of votes is chosen.
- No cost to vote for an option that is unlikely to win.

Individual	1	2	3	4	5
	a	a	a	b	c
	b	b	b	a	b
. . . disapproves	c	c	c	c	a

Alternative a is the Condorcet winner, but b is selected using approval voting.

Runoff voting

- Each voter selects only one option, and a second runoff election takes place between the two strongest alternatives if there is no majority in the first place.
- Widely used.

Number of individuals	6	5	4	2
	a	c	b	b
	b	a	c	a
	c	b	a	c

Options *a* and *b* survive the first round. In the runoff, option *a* wins over *b*.

But, assume option *a* attracts more partisans:

Number of individuals	6	5	4	2
	a	c	b	a $\uparrow$
	b	a	c	b $\downarrow$
	c	b	a	c

Options *a* and *c* survive the first round. In the runoff, option *a* looses over *c* despite having gained supporters.

Majority judgment

- Voters evaluate every candidate using (ordinal) grades. Candidates are *judged*, not compared.
- Final *majority-grade* of each candidate is his or her median grade.
- The *majority-ranking* orders candidates according to their majority-grades.
- Shown to solve most of problems raised by Arrow. In particular, it is more robust than other rules to strategic voting.

Michel Balinski & Rida Laraki, 2010. "Election by Majority Judgement: Experimental Evidence," Chapter in the book: In Situ and Laboratory Experiments on Electoral Law Reform: French Presidential Elections. Springer.

- French Presidential election of 2007.
- First round (April 22, 2007).
- Field experiment in three (out of 12) precincts of Orsay.
- $1,733$ participants (74% of voters).

Table 2.10 Majority judgment results, three precincts of Orsay, April 22, 2007

	Excellent	*Very Good*	*Good*	*Acceptable*	*Poor*	*to Reject*
Besancenot	4.1%	9.9%	16.3%	16.0%	22.6%	31.1%
Buffet	2.5%	7.6%	12.5%	20.6%	26.4%	30.4%
Schivardi	0.5%	1.0%	3.9%	9.5%	24.9%	60.4%
Bayrou	13.6%	30.7%	25.1%	14.8%	8.4%	7.4%
Bové	1.5%	6.0%	11.4%	16.0%	25.7%	39.5%
Voynet	2.9%	9.3%	17.5%	23.7%	26.1%	20.5%
Villiers	2.4%	6.4%	8.7%	11.3%	15.8%	55.5%
Royal	16.7%	22.7%	19.1%	16.8%	12.2%	12.6%
Nihous	0.3%	1.8%	5.3%	11.0%	26.7%	55.0%
Le Pen	3.0%	4.6%	6.2%	6.5%	5.4%	74.4%
Laguiller	2.1%	5.3%	10.2%	16.6%	25.9%	40.1%
Sarkozy	19.1%	19.8%	14.3%	11.5%	7.1%	28.2%

Source: Balinski and Laraki (2010)

Table 2.11 The majority-gauges (p, α, q) and the majority-ranking, three precincts of Orsay, April 22, 2007

	Majority-ranking	p = Above maj.-grade	α^* = The majority-grade*	q = Below maj.-grade	Natl. rank.	Orsay rank.
1st	Bayrou	*44.3%*	*Good*$^+$	30.6%	3rd	3rd
2nd	Royal	39.4%	*Good*$^-$	*41.5%*	2nd	1st
3rd	Sarkozy	38.9%	*Good*$^-$	*46.9%*	1st	2nd
4th	Voynet	29.8%	*Acceptable*$^-$	*46.6%*	8th	7th
5th	Besancenot	*46.3%*	*Poor*$^+$	31.2%	5th	5th
6th	Buffet	*43.2%*	*Poor*$^+$	30.5%	7th	8th
7th	Bové	34.9%	*Poor*$^-$	*39.4%*	10th	9th
8th	Laguiller	34.2%	*Poor*$^-$	*40.0%*	9th	10th
9th	Nihous	45.0%	*to Reject*	–	11th	11th
10th	Villiers	44.5%	*to Reject*	–	6th	6th
11th	Schivardi	39.7%	*to Reject*	–	12th	12th
12th	Le Pen	25.7%	*to Reject*	–	4th	4th

The columns headed "Natl. rank." and "Orsay rank." are the national rank-orders by the current system

Source: Balinski and Laraki (2010)

Comments

- Majority judgment does not lead to the same outcome as runoff voting.
- Unsurprisingly, majority judgment correctly "predicts" the outcome in face-to-face confrontation: On May 6, 2007, S. Royal beats N. Sarkozy in Orsay (51.3% vs. 48.7%).
- Majority judgment incites candidates so receive the highest possible evaluation from every voter – not only to seduce 51% of voters –, what give more weight to minorities.

Conclusions on voting

- Other "problems" appear when considering the decision to vote or proportional representation of population.
- All rules have drawbacks in the sense that they violate one or more of Arrow's conditions. This is inevitable. Whatever scheme we choose will have some problem associated with it.

Social welfare functions

- A social welfare function allows to evaluate or compare economic policies that cause redistribution between consumers.
- How to decide whether things are going better or worse? How to compare situations across space and time?
- These are questions asked to researchers, policy makers, and pub regulars.

Pareto and quasi-Pareto criteria

- Pareto improvement:

  Somebody is made better off and nobody else is made worse off.

  Need to know each individual utility to use it.

- Quasi-Pareto improvement:

  Somebody's real income does up and nobody's real income goes down.

  Much more practical.

Paretian welfare functions

- Welfaristic social welfare function:

  A social welfare function is welfaristic if its arguments are the utilities of the various individuals, i.e.

  $$\mathbb{W} = f\left(U_1, \ldots, U_n\right),$$

  and only the utilities of individuals enter the social welfare function.

  Also called *individualistic* function.

- Paretian social welfare function:

  A social welfare function is Paretian if it approves any Pareto improvement. Equivalently, if it judges any Pareto-superior state to be better than a Pareto-inferior state:

$$\mathbb{W} = f\left(U_1, \ldots, U_n\right) \text{ and } \frac{\partial \mathbb{W}}{\partial U_i} > 0, \forall i.$$

All Paretian social welfare functions are individualistic.

Examples

- Bergsonian social welfare function:

$$\mathbb{W} = a_1 U_1 + a_2 U_2 + \ldots + a_n U_n, \text{ with } a_i > 0 \forall i.$$

- Utilitarian (or Benthamite) social welfare function:

$$\mathbb{W} = aU_1 + U_2 + \ldots + U_n.$$

Non-Paretian social welfare function

An individualistic social function may be non-Paretian. For example, an *observer's social welfare function* where the observer cares about something different from what individuals care about. In a two-individual economy, such a function could be an egalitarian function such as:

$$\mathbb{W} = f\left(|U_1 - U_2|\right), \text{ with } f' < 0.$$

In this case, the observer cares about $|U_1 - U_2|$.

A social welfare function may be non-Paretian because it does not judge all Pareto improvement to be strictly preferable. This is the case of the Rawlsian social welfare function:

$$\mathbb{W} = f\left[\min_i (U_i)\right], \text{ with } f' > 0.$$

Example: $(5,4,1) \succ (5,3,1)$ in the sense of Pareto, but not from the rawlsian point of view.

But, the lexicographic Rawlsian social welfare function is Paretian: If the poorest individual's utility is unchanged, look at the next poorest individual's utility, and so until you find a change. If that individual's utility has increased, social welfare goes up. By this criterion, any Pareto improvement will be judged to be welfare-increasing. Example: $(5, 4, 1) \succ (5, 3, 1)$ from the lexicographic rawlsian point of view, but not from the rawlsian point of view.

Non-individualistic social welfare functions

A social welfare function is non-individualistic if it is not a function of the utilities of the individuals, i.e. it does not accept their preferences.

Why might an observer not want to accept individuals' preferences? Example in a two-person economy where individuals exhibit "envy":

$$U_1 = g(y_1, y_2), \text{ with } \frac{\partial g}{\partial y_1} > 0, \text{ and } \frac{\partial g}{\partial y_2} < 0,$$
$$U_2 = h(y_1, y_2), \text{ with } \frac{\partial h}{\partial y_2} > 0, \text{ and } \frac{\partial h}{\partial y_1} < 0.$$

The observer may prefer to use a function that is monotonic in each individual's income:

$$\mathbb{W} = f(v_1(y_1), v_2(y_2)), \text{ with } \frac{\partial v_i}{\partial y_i} > 0 \text{ and } \frac{\partial \mathbb{W}}{\partial v_i(y_i)} > 0.$$

Abbreviated social welfare function

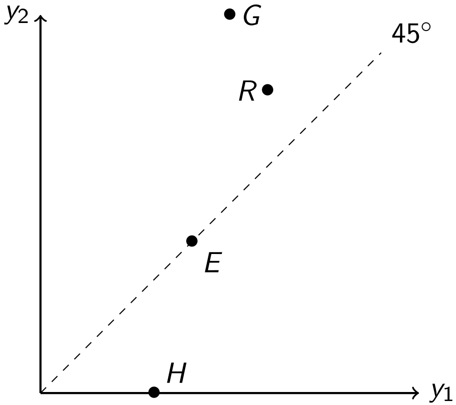
A social welfare function is abbreviated if welfare is expressed as a function of statistics calculated from the income distribution vector.

Example:

$$\mathbb{W} = f\left(\text{Production, Inequality, Poverty}\right),$$

with (usually), $\frac{\partial g}{\partial \text{Production}} > 0$, $\frac{\partial g}{\partial \text{Inequality}} < 0$, and $\frac{\partial g}{\partial \text{Poverty}} < 0$.

Disagreements among approaches



Which is the preferred income distribution?

- $\mathbb{W} = f\,(\text{Production})$:
  $G \succ R \succ E \succ H$.
- $\mathbb{W} = f\,(\text{Equality})$:
  $E \succ R \succ H \succ G$.
- $\mathbb{W} =$ Rawlsian criterion:
  $R \succ G \succ E \succ H$.

End of lecture.

Lectures of this course are inspired from those taught by R. Chetty, G. Fields, N. Gravel, H. Hoynes, and E. Saez.