Public Economics

Optional intermediary exam

Marc Sangnier - marc.sangnier@univ-amu.fr

February 7th, 2013

The exam lasts 90 minutes. Documents are not allowed. You can answer either in French or in English. Answer questions below and 3 out of the 4 exercises. Please indicate clearly the exercises you choose.

> Questions and exercises inspired from *Intermediate Public Economics*, by J. Hindriks and G.D. Myles.

Questions 5 points

Answer briefly (3 or 4 lines per question) to the following questions.

1. Why is income more taxed than wealth? 1

> Taxation requires information. Income is a flow that the government can observe without difficulty for much of the population (e.g. rely on employers to report information on incomes of employees). In contrast, wealth is a stock that is held in widely diverse forms. A monetary value can be placed directly on some forms of wealth such as bank deposits or share holdings. Other forms, such as housing can be valued with more uncertainty. Finally, there are other forms of wealth, such as works of art and jewelry, that are even more difficult to value.

2. Discuss how television technology can turn a public good into a private good. 1.5

> Television has long been broadcast as an analog signal from local transmitters. Such signals are accessible to anyone equipped with suitable equipment. As such, it is not possible to easily exclude someone from television consumption. This is one of the characteristics of a public good. The introduction of transmission through digital signals has made simpler to control access to the signal as subscription services require the use of a card or an access number to decode the signal. Exclusion is now feasible at low cost, what turned television signal from into a private good.

3. Are the following statements true or false? Explain very briefly why. 0.5×5

   a) If the supply of public good is determined by majority vote, then the outcome must be Pareto-efficient.

      False. According to the median voter theorem, the chosen level of public good will be the one preferred by the median voter. Nothing guarantees *a priori* that this will be the socially efficient level.

   b) If preferences are single-peaked, then everyone will agree about the right amount of public goods to be supplied.

      False. The assumption of single-peaked preferences simply means that each individual has a single most-preferred quantity. These preferred quantity can differ among individuals.

   c) Public goods are those goods that are supplied by the government.

      False. Public goods are defined by non-rivalry and non-excludability. They are often supplied by the government but may also be supplied by other actors.

   d) If a public good is provided by voluntary contributions, too little will be supplied relative to the efficient level.

      True. Private choices ignore the social benefits of additional provision. An increase in contributions would lead to a Pareto-improvement.

   e) The theory of optimal commodity taxation argues that equal tax rates should be set across all commodities so as to maximize efficiency by "smoothing taxes".

      False. Theory of optimal taxation makes clear that taxes should be differentiated between commodities in order to minimize the taxation burden. In particular, a commodity with a low elasticity of demand should be taxed at a higher rate than a commodity with a high elasticity of demand.

Exercise 1

5 points

Let there be N identical consumers indexed by $i = 1, \dots, N$. Each of them has the same utility function:

$$U^i = \log(x^i) + \log(G),$$

where x^i is the consumption of a private good by individual i, and G is a pure public good. Each consumer is endowed with income 1. Let 1 be the unit price of the private good, such that each consumer's budget constraint can be written as:

$$x^i + g^i \leq 1,$$

where g^i is individuals i's contribution to the public good. Total available quantity of the public good is the sum of individual contributions, i.e.:

$$G = \sum_{i=1}^{N} g^i.$$

1. Calculate G^d, the equilibrium public good provision when individuals take decentralized decisions. 2

Given the budget constraint and the production function of the public good, individual i chooses g^i in order to maximize:

$$U^i = \log(1 - g^i) + \log\left(g^i + \sum_{j=1, j\neq i}^{N} g^j\right).$$

The first order condition with respect to g^i is:

$$\frac{\partial U^i}{\partial g^i} = -\frac{1}{1 - g^i} + \frac{1}{g^i + \sum_{j=1,j\neq i}^{N} g^j} = 0.$$

Since all individuals are identical, we know that $\forall i$, $g^i = g$. The above condition can be rewritten as:

$$\frac{1}{Ng} = \frac{1}{1-g} \Leftrightarrow g = \frac{1}{N+1}.$$

Thus, total public good provision is:

$$G^d = Ng = \frac{N}{N+1}.$$

2. Calculate G^o, the optimum public good provision when a social planner chooses the level of public good such has each individual contributes equally and the following social welfare function is maximized: 2

$$\mathbb{W} = \sum_{i=1}^{N} U^i.$$

The social planner chooses G in order to maximizes the following expression:

$$\mathbb{W} = \sum_{i=1}^{N} \left[\log(1 - \frac{G}{N}) + \log(G)\right] = N\left[\log(1 - \frac{G}{N}) + \log(G)\right].$$

The associated first order condition is:

$$\frac{\partial \mathbb{W}}{\partial G} = N\left[-\frac{1}{1-\frac{G}{N}}\frac{1}{N} + \frac{1}{G}\right] = 0,$$

which gives:

$$G^o = \frac{N}{2}.$$

3. Comment on the effect of changing N on the difference between the decentralized equilibrium and the social optimum. 1

As N increases, the gap between G^o and G^d increases. This effect arises from the fact that as each individual contribution gains social value as it benefits to more and more individuals as the population increases.

Exercise 2

5 points

A competitive refining industry produces a refined product. The inverse demand function for the refined product is:

$$p^d = 20 - q,$$

where q is the quantity consumed when consumers pay price p^d. The inverse supply curve for the refined product is:

$$p^s = 2 + q,$$

when the industry produces q units sold at price p^s.

1. What are the market equilibrium price and quantity for the refined product? 1

The market equilibrium price p^* is such that $p^d = p^= p^s$, that is:

$$20 - q = 2 + q \Leftrightarrow q^* = 9 \text{ and } p^* = 11.$$

The industry releases one unit of waste into the atmosphere for each unit of refined product. The marginal cost of pollution is:

$$\text{MC} = q,$$

when the industry releases q units of waste.

2. Express the social marginal cost of production. 1

The social marginal cost of production is the sum of marginal cost of production p^s and the marginal cost of pollution MC:

$$2 + q + q = 2 + 2q.$$

3. Calculate the price and quantity for the refined product at the social optimum. 1.5

At the social optimum, we must have equality between social marginal cost and social marginal benefit p^d, that is:

$$2 + 2q = 20 - q \Leftrightarrow q^o = 6 \text{ and } p^o = 14.$$

4. Assume that the government imposes an emission fee of T per unit of emissions. How large must the emission fee be to let the market produce the socially efficient amount of the refined product? 1.5

With an emission fee of T per unit, inverse supply function of the industry becomes:

$$p^s(T) = 2 + q + T.$$

The market equilibrium will thus be such that $p^s(T) = p^d$, that is:

$$2 + q + T = 20 - q \Leftrightarrow q(T) = 9 - \frac{T}{2}.$$

The government must set T such that $q(T) = q^o$, that is:

$$9 - \frac{T}{2} = 6 \Leftrightarrow T = 6.$$

Exercise 3

5 points

Consider a society made of three individuals indexed by A, B, and C. Let $G \in [0, +\infty[$ be the number of hours of television broadcast each day. Television broadcast is financed through a tax shared equally among individuals, i.e. if G is supplied, each individual has to pay $\frac{G}{3}$. Assume the individuals have the following before tax utilities over G:

$$U^A = G,$$
$$U^B = 2 - G,$$
$$U^C = \frac{4}{3}G - \frac{G^2}{2},$$

and that these utilities can be directly compared to the tax cost.

1. Show that the three individuals have single-peaked preferences. 1.5

> Utility of individual A is strictly increasing with G, thus there is an infinite quantity that this individual prefers to all others below. Utility of individual B is strictly decreasing with G, thus he will prefer a quantity 0 of the public good to any other quantity above it. Utility of individual C is maximum when $G = \frac{4}{3}$. It is strictly decreasing when moving away from this level.

2. If the government is choosing G from the range $0 \leq G \leq 2$, what is the majority voting outcome G^v? 2

> Individual A preferred quantity of the public good is $G^A = 2$. Individual B prefers $G^B = 0$. Individual C chooses its preferred quantity in order to maximize:
> $$U^C - \frac{G}{3} = G - \frac{G^2}{2},$$
> that is $G^C = 1$. Since $G^B < G^C < G^A$ and since preferences of the three individuals are single-peaked, the median voter theorem can be applied. Accordingly, the quantity chosen by majority voting will be $G^v = G^C = 1$.

Aggregate social welfare can be expressed as:

$$\mathbb{W} = U^A + U^B + U^C - G.$$

3. Does the majority voting outcome maximize social welfare? Comment. 1.5

> Let us choose quantity G that maximizes $\mathbb{W}$:
> $$\frac{\partial \mathbb{W}}{\partial G} = 0 \Leftrightarrow G^{\mathbb{W}} = \frac{1}{3} \neq G^v.$$
> This illustrates the fact that nothing guarantees *a priori* that majority voting will maximize social welfare.

Exercise 4 5 points

Total tax revenue raised by a government is given by:

$$\mathbb{R}(t) = t \times \mathbb{B}(t),$$

where $t \in [0, 1]$ is the tax rate and $\mathbb{B}(t)$ is the tax base, with $\frac{\partial \mathbb{B}}{\partial t} < 0$. Suppose that the elasticity of the tax base can be expressed as:

$$\varepsilon = -\frac{\gamma t}{1 - \gamma t},$$

with $\gamma \in [0, 1]$.

1. Explain what is the elasticity of the tax base. 1

The elasticity of the tax base measures the change in tax base induced by a change in tax rate. It is a ratio of percentages:

$$\varepsilon = \frac{d\mathbb{B}/\mathbb{B}}{dt/t} = \frac{t}{\mathbb{B}}\frac{d\mathbb{B}}{dt}.$$

2. What is the tax rate that maximizes total tax revenue? Let us call it t^*. 2

The necessary condition to maximize total tax revenue with respect to tax rate is:

$$\frac{\partial \mathbb{R}}{\partial t} = 0 \Leftrightarrow \mathbb{B}(t) + t\frac{\partial \mathbb{B}}{\partial t} = 0.$$

Given the expression of ε, this condition can be rewritten as:

$$\begin{aligned} \mathbb{B} + \tfrac{\mathbb{B}}{\mathbb{B}} t \tfrac{\partial \mathbb{B}}{\partial t} = 0 \quad &\Leftrightarrow \mathbb{B} + \mathbb{B}\varepsilon = 0 \\ &\Leftrightarrow \mathbb{B}\left[1 + \varepsilon\right] = 0. \end{aligned}$$

Thus, t^* satisfies:

$$\begin{aligned} \varepsilon + 1 = 0 \quad &\Leftrightarrow \tfrac{\gamma t}{1-\gamma t} = 1 \\ &\Leftrightarrow t^* = \tfrac{1}{2\gamma}. \end{aligned}$$

3. How does t^* vary with γ? Explain the intuition behind it. 2

t^* decreases with γ. This arises from the fact that the absolute value of the elasticity of the tax base with respect to tax rate is increasing with γ. Thus, the higher γ, the larger the reduction in tax base induced by an increase in tax rate. As a consequence, the optimal tax rate is lower when the tax base is more sensitive to taxes.