

# Public Economics

## Final exam

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The exam lasts 90 minutes. Documents are not allowed. The use of a calculator is allowed. Any other electronic devices are forbidden. You can answer either in French or in English. Answer the *two exercises* and *one out of the two questions* below.

Exercise 1 and question 1 are inspired from *Intermediate Public Economics*, by J. Hindriks and G.D. Myles. Exercise 2 and question 2 are inspired from problem sets by J. Gruber.

### Exercise 1

6 points

Consider a large population of commuters. They can individually decide to use either their car or the train to commute. Commuting by train takes 70 minutes whatever the number of persons taking the train. Commuting by car takes  $C(x) = 20 + 60x$  minutes, where  $x$  is the proportion of commuters taking their car,  $0 \leq x \leq 1$ .

1. Explain intuitively why travel time by car varies with  $x$ .

1

Travel time by car increase with the proportion of commuters that used their car because of congestion. Each additional driver imposes an externality on other drivers, driving speed is reduced and travel time increases.

2. Show that, if everyone is taking her decision freely and independently so as to minimize her own commuting time, the equilibrium proportion of commuters who will travel by car is  $x_m = \frac{5}{6}$ .

*Hint:* Each individual chooses the mode of transport that has the lowest commuting time given  $x$ . In equilibrium, individuals are indifferent between the train and the car.

1

In the equilibrium situation, each commuter will be indifferent to travel by car or by train, i.e. both travel times will be equal. Formally, the decentralized equilibrium proportion of car users  $x_m$  is such that:

$$70 = 20 + 60x_m \Leftrightarrow x_m = \frac{5}{6}.$$

3. Show that the proportion of car users that minimizes the total (or average) commuting time is  $x_e = \frac{5}{12}$ . 1

Total commuting time  $T$  can be written as:

$$T = (20 + 60x)x + 70(1 - x).$$

It is minimal when  $\frac{\partial T}{\partial x} = 0$ , that is:

$$x_e = \frac{5}{12}.$$

4. Compare answers to the two previous questions. Why do they differ? How large is the deadweight loss? 1

It is straightforward to see that  $x_e < x_m$ . The proportion of drivers when everyone freely decides how to travel exceed the social optimum because of the negative externality that each driver imposes on others. This externality is not taken into account by individuals when they choose individually. The social deadweight loss from the externality is the difference in total commuting time between both situations. Under the market equilibrium, total commuting time is  $T_m = (20 + 60x_m)x_m + 70(1 - x_m) = \frac{840}{12}$ . At the social optimum, total commuting time is  $T_e = (20 + 60x_e)x_e + 70(1 - x_e) = \frac{715}{12}$ . Thus, the deadweight loss is  $T_m - T_e = \frac{125}{12}$  minutes.

5. Assume that commuters value their time as if 1 minute is priced 1 euro. How could a toll help to achieve social optimality? What fare should be charged on car users? 2

If commuters value time spent traveling, then a toll can help to achieve social optimality by increasing cost of commuting by car. By setting the correct fare, one may reach a situation where the proportion of commuters that still find it beneficial to travel by car is exactly equal to the socially optimal level  $x_e$ . Let us look for  $\tau$  such that:

$$\tau + 20 + 60\frac{5}{12} = 70 \Leftrightarrow \tau = 25.$$



**Exercise 2**

8 points

Physicians choose the level of care  $x$  they provide to each of their patients. For the doctor, the cost (translated in monetary units) of providing  $x$  units of care is  $C(x) = 5x^2 + 5$ . For a representative patient, the benefit (translated in monetary units) of receiving  $x$  units of care is  $B(x) = 90x - 10x^2$ .

1. Define the socially optimal level of care  $x^*$ . Show that  $x^* = 3$ .

0.5

The socially optimal level of care is the level of care for which the social marginal benefit equals the social marginal cost. Here, the social marginal benefit is the marginal health improvement for patients, i.e.  $\frac{\partial B(x)}{\partial x}$ , and the social marginal cost is the marginal cost suffered by physicians, i.e.  $\frac{\partial C(x)}{\partial x}$ . Equality between these two terms immediately yields  $x^* = 3$ .

Let us assume that physicians have the following utility function:

$$U(x) = (1 - \lambda)P(x) + \lambda B(x) - C(x),$$

where  $\lambda \in [0, 1]$ , and  $P(x)$  is the monetary payment received by the doctor from a third party—e.g. the state—when she provides  $x$  units of care to a patient.

2. What does  $\lambda$  represent?

0.5

Parameter  $\lambda$  represents the weight put by the doctor on health benefits for patients rather than in her private monetary payoff. If  $\lambda = 0$ , physicians are pure profit maximizers. In contrast,  $\lambda = 1$  corresponds to the case where physicians disregard monetary revenues and only care about patients' health.

Suppose first that there is no health system and that physicians are not paid when they provide care, i.e.  $P(x) = 0$ .

3. What will be the level of care  $x^a$  provided by physicians?

1

Each doctor maximizes  $\lambda B(x) - C(x)$ . This yields:

$$\lambda(90 - 20x) = 10x \Leftrightarrow x^a = \frac{90\lambda}{20\lambda + 10}.$$

4. Discuss how  $x^a$  varies with  $\lambda$ .

1

$x^a$  increases with  $\lambda$  and is equal to  $x^*$  when  $\lambda = 1$ . In that case, physicians fully internalize social benefits of cares and completely disregard monetary incentives.

From now on, assume that  $\lambda < 1$ . Let us suppose that the government want to use retrospective payments to pay physicians. Under such a setting, physicians choose the level of  $x$  and received  $K$  euros per unit of care provided, that is  $P(x) = Kx$ .

5. What will be the level of care  $x^r$  under this setting? 1

The physician choose the level of care that maximizes its utility function, that is:

$$\begin{aligned} \frac{\partial U(x)}{\partial x} = 0 &\Leftrightarrow (1 - \lambda)K + \lambda(90 - 20x) = 10x \\ &\Leftrightarrow x^r = \frac{K + (90 - K)\lambda}{10 + 20\lambda}. \end{aligned}$$

6. Compute the amount  $K^*$  that ensures that  $x^r = x^*$ . 1

Let us look for  $K$  such that  $x^r = 3$ .

$$\begin{aligned} \frac{K + (90 - K)\lambda}{10 + 20\lambda} &= 3 \\ \Leftrightarrow K^* &= 30. \end{aligned}$$

$K^*$  is a decreasing function of  $\lambda$ . For example, if  $\lambda = 0$ , then  $K^* = 30$ . And as  $\lambda$  goes to 1, then  $K^*$

7. What happens if the government sets  $K \neq K^*$ ?  
*Note:* Assume that, for some reason,  $K$  cannot exceed 90. 1

As  $x^r$  is an increasing function of  $K$ , there will be over-provision of care if the government set  $K > K^*$ . In contrast, there will be under-provision if the government sets  $K < K^*$ .

Assume now that the government uses a prospective payment system. Under such a scheme, a physician receives a given amount  $K_p$  per patient treated if at least  $\bar{x}$  of care have been provided. In other words,  $P(x) = K_p$  if  $x \geq \bar{x}$ , and  $P(x) = 0$  if  $x < \bar{x}$ .

8. What are the values of  $\bar{x}$  and  $K_p$  that the government needs to choose to ensure that physicians provide the socially optimal level of care  $x^*$ ?  
*Hint:* The government must set  $\bar{x}$  and  $K_p$  such that physicians have incentives to provide care, and they provide the required level of care. 1

The government can simply set  $\bar{x} = 3$ , such that physicians are paid if and only if they provide the socially efficient level of care. Now, the government must set  $K_p$  such that physicians prefer to provide  $\bar{x} = 3$  and to receive the payment, rather than not to provide any care. In other words, the representative physician must be better off when she provides  $x = 3$  than  $x = 0$ . This

condition can be written as:

$$\begin{aligned} (1 - \lambda) P(3) + \lambda B(3) - C(3) &> (1 - \lambda) P(0) + \lambda B(0) - C(0) \\ \Leftrightarrow (1 - \lambda) K_p + \lambda (270 - 90) - (45 + 5) &> 0 \\ \Leftrightarrow K_p > \frac{1 - \lambda}{4\lambda}. \end{aligned}$$

Thus, the government can set  $\bar{x} = 3$  and  $K_p = \frac{1 - \lambda}{4\lambda}$ .

9. Discuss whether the government should use a prospective payment system or retrospective payments depending on  $\lambda$ .

1

Here, both systems allow to achieve the socially optimal level of care. Under the retrospective payment system, the total bill equals  $K_r \times x^* = 30 \times 3 = 90$  euros. Under the prospective payment system, the total bill equals  $K_p = \frac{1 - \lambda}{4\lambda}$ . The latter number is larger than the former if and only if  $\lambda < \frac{1}{361}$ , i.e. if physicians place a virtually null value on patients health. So, the government should choose the retrospective payment system, except if  $\lambda$  is extremely low. In the limit case where  $\lambda = 0$ , the government should choose the prospective payment system.

### Question 1

6 points

Consider a country where the largest part of health insurance is provided by private firms—e.g. the United States. Some health insurance companies would like to use genetic testing to have more information about the health status of their applicants. Should the government allow them to act so?

*Hint:* You might want to think about the following sub-questions. Would genetic testing help or hurt those who have bad health prospects? Would it help or hurt those who are have good health prospects? Would it exacerbate or mitigate the problem of adverse selection in the health insurance market? Would it increase the number of people without health insurance?

Health insurance is a classical example of adverse selection. If such insurance is provided by private companies, it is likely that the equilibrium is a separating equilibrium where low-risk and high-risk individuals face different contracts. The equilibrium is such that low risk individuals—those in good health—are under insured such as to prevent high-risk individuals—those in bad health—to buy the same contract. The price difference between contract allow those in good health to signal their health status—unobservable by companies. Hence, genetic testing would be beneficial for low risk individuals as this will reveal their health status. This improvement will occur at no cost for those in bad health. All in all, genetic testing would mitigate the problem of adverse selection. However, under a pooling equilibrium—only one contract is offered to all individuals—, those in good health implicitly subsidize

those in bad health. As such, genetic testing would allow to personalize prizes and may push those in bad health out of the market—leaving them with no insurance at all. As a general result, the government should takes its decision to allow insurance companies to use genetic testing depending on the initial situation of the health insurance market: under a separating equilibrium, participation rate won't change but low-risk individuals might be better off; under a pooling, participation rate of high-risk individuals might drop at the benefit of low-risk individuals that might be better off.

**Question 2**

6 points

You have been hired by the PACA region to evaluate a reform. This reform imposes harsh training requirements on those who receive unemployment benefits. These requirements may both increase employability of unemployed people and make unemployment benefits less “attractive” for cheaters. The local authorities would like to know whether this reform induced individuals to increase their labor supply, and hence earn more from labor income. This reform has been applied in 2012 to single male individuals living in PACA region. For your evaluation, your are provided with the following information on average weekly earning in euros of male residents living in PACA and Aquitaine regions.

Region	Year	Marital status	Labor income
PACA	2011	Single	170
PACA	2011	Married	200
PACA	2012	Single	210
PACA	2012	Married	230
Aquitaine	2011	Single	220
Aquitaine	2011	Married	240
Aquitaine	2012	Single	240
Aquitaine	2012	Married	270

Propose two difference-in-differences estimators of the impact of the reform. For each of them, give the assumption required for the estimator to be valid, and compute the value of the estimator. Finally, discuss which one you would consider as more valid.

*Note:* This question and the numbers provided are purely imaginary.

The treatment period is 2012. The pre-treatment period is 2011. The treated group is made of single male individuals living in PACA region. There might be two candidate groups for the control group: (i) under the assumption that single male individuals behave as married male individuals living in the same region, married male individuals living in the PACA region represent a valid control group; (ii) under the assumption that single male individuals living in another region behave

as single male individuals living in the PACA region, single male individual living in the Aquitaine region represent a valid control group. Thus, we can compute two difference-in-differences estimators:

$$\begin{aligned}\beta_i &= (\text{Single PACA after} - \text{Single PACA before}) \\ &\quad - (\text{Married PACA after} - \text{Married PACA before}) \\ &= (210 - 170) - (230 - 200) \\ &= 40 - 30 \\ &= 10,\end{aligned}$$

and

$$\begin{aligned}\beta_{ii} &= (\text{Single PACA after} - \text{Single PACA before}) \\ &\quad - (\text{Single Aquitaine after} - \text{Single Aquitaine before}) \\ &= (210 - 170) - (240 - 220) \\ &= 40 - 20 \\ &= 20.\end{aligned}$$

Both estimator give a positive effect for the reform—an increase of 10 or 20 euros in average weekly earnings. The first estimator relies on the strong assumption that married and single individuals would have behave similarly in the absence of the reform. However, married and single individuals are likely to be radically different. The second estimator gets rid of this assumption but assumes that there are no shocks that are specific to PACA and Aquitaine regions, what is likely to be false.

There is no ultimate way to say which of these two estimator is the best. However, it is possible to propose a combined estimator that would take advantage of both approaches. Let us compare the difference in the evolution of treated individuals with respect to married individuals living in the PACA region and the difference in the evolution of single individuals and married individuals living in the Aquitaine region. This difference-in-differences-in-differences estimator can be written as follows:

$$\begin{aligned}\beta_{iii} &= \{(\text{Single PACA after} - \text{Single PACA before}) \\ &\quad - (\text{Married PACA after} - \text{Married PACA before})\} \\ &\quad - \{(\text{Single Aquitaine after} - \text{Single Aquitaine before}) \\ &\quad - (\text{Married Aquitaine after} - \text{Married Aquitaine before})\} \\ &= \{(210 - 170) - (230 - 200)\} - \{(240 - 220) - (270 - 240)\} \\ &= \{40 - 30\} - \{20 - 30\} \\ &= 20.\end{aligned}$$