Public Economics

Optional intermediary exam

Solutions

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February 13th, 2015

The exam lasts 90 minutes. Documents are not allowed. The use of a calculator or of any other electronic devices is not allowed. You can answer either in French or in English.

> Exercises 1 and 2 are inspired from *Intermediate Public Economics*, by J. Hindriks and G.D. Myles. Exercise 3 is inspired from *Microeconomic Theory*, by A. Mas-Colell and M.D. Whinston.

Exercise 1 — 8 points

In a *transferable voting system* each voter provides a ranking of options. If no option achieves the majority, the option with the lowest number of first-choice votes is eliminated and the votes that were attached to it are transfered to the second-choice options (for voters who first-choice was eliminated). This process proceeds until an option achieves a majority.

1. Define what is a Condorcet winner. 1

> The Condorcet winner is the alternative that defeats all other alternatives in pairwise majority voting.

2. Is it possible for an option that is no one's first choice to win under a transferable voting system? 2

> No. A Candidate that is no one's first choice will be eliminated in the first round under a transferable voting system as the candidate with the lowest number of first-choice votes (i.e., zero). Furthermore, is an option achieves the majority at the first round, it cannot be an option that is no one's first choice.

Consider the following preferences of five voters $i = 1, \ldots, 5$ over three alternatives a, b, and c:

	1	2	3	4	5
Most preferred alternative	a	b	b	c	c
	b	a	a	a	a
Least preferred alternative	c	c	c	b	b

3. Assume that voters truly express their preferences. What will be the selected option under a transferable voting system? Is this the Condorcet winner? 2

> No option achieves the majority at the first round. Thus, option a will be eliminated as it is the first-choice of only one voter. Preferences can now be represented as:
>
> | | 1 | 2 | 3 | 4 | 5 |
> |---|---|---|---|---|---|
> | Most preferred alternative | b | b | b | c | c |
> | Least preferred alternative | c | c | c | b | b |
>
> Now, b achieves the majority. This will be the selected option. Note that this is not the Condorcet winner. Indeed, a is the Condorcet winner.

4. Show how strategic voting can affect the outcome of the vote. What will be the outcome of the vote if voters vote strategically? 3

> Let us check whether individuals have incentives not to truly express their preferences.
>
> Individual 1 has no incentives to vote differently as she cannot affect the outcome of the vote in a direction she would prefer. Obviously, individuals 2 and 3 have no incentives not to vote sincerely as their preferred option will be the selected one.
>
> In contrast, voters 4 and 5 would be better off under c or a than under b which is their least preferred option. Actually, they can affect the outcome of the vote by choosing to vote for a rather than for c in the first round. This would allow a to achieve the majority and to be the selected option. Individuals 4 and 5 would then be better off than under b.
>
> This illustrates that if there is strategic voting, a transferable voting system selects the Condorcert winner.

Exercise 2 6 points

Let us consider an economy populated by 2 individuals—A and B—who consume 2 goods—1 and 2. Individuals' utility function are:

$$U^A = \log(x_1^A) + \log(x_2^A) + \frac{1}{2}\log(x_1^B),$$

and

$$U^B = \log(x_1^B) + \log(x_2^B) + \frac{1}{2}\log(x_1^A),$$

where x_j^i is the quantity of good j consumed by individual i. Each individual is endowed with 1 unit of income. Let the unit prices of both goods be 1.

1. Calculate the decentralized equilibrium situation of this economy. 1

> Each individual maximizes her utility function subject to her budget constraint. Accordingly, the Lagrangian of individual i's optimization problem is:
> $$\mathbb{L} = U^i = \log(x_1^i) + \log(x_2^i) + \frac{1}{2}\log(x_1^{-i}) + \lambda_i(1 - x_1^i - x_2^i),$$
> where x_1^{-i} denotes consumption of good 1 by the other consumer. Solving this program for each individual yields:
> $$x_1^A = \frac{1}{2} \text{ and } x_2^A = \frac{1}{2},$$
> $$x_1^B = \frac{1}{2} \text{ and } x_2^B = \frac{1}{2}.$$

2. Calculate the social optimum if the social welfare function is the sum of individuals' utility functions. 1

> Let us maximize $\mathbb{W} = U^A + U^B$ with respect to x_1^A, x_1^B, x_2^A, and x_2^B, subject to $x_1^A + x_2^B \leq 1$ and $x_1^B + x_2^B \leq 1$. We get :
> $$x_1^A = \frac{3}{5} \text{ and } x_2^A = \frac{2}{5},$$
> $$x_1^B = \frac{3}{5} \text{ and } x_2^B = \frac{2}{5}.$$

3. Compare quantities of good 1 under both situations. Comment. 2

> At the decentralized equilibrium total quantity of good 1 is lower than at the social optimum. This comes from the fact that, when maximizing their own utility, individuals do not take into account the positive externality that is associated with the consumption of good 1. This result in an under-provision of good 1.

4. Show that the social optimum can be reached in a decentralized framework thanks to a subsidy s placed on good 1 (so, the price of this good is now $1 - s$), with the cost of this subsidy covered by a lump-sum tax T on each consumer. 2

> Individual i's Lagrangian should now be written as:
> $$\mathbb{L} = U^i = \log(x_1^i) + \log(x_2^i) + \frac{1}{2}\log(x_1^{-i}) + \lambda_i\left[1 - T - (1-s)x_1^i - x_2^i\right].$$
> Solving yields:
> $$x_1^i = \frac{1-T}{2(1-s)}.$$

Since we want $x_1^i = \frac{3}{5}$, we know that per capita subsidy cost will be $T = s\frac{3}{5}$. We just need to solve the following expression:

$$\frac{1-T}{2(1-s)} = \frac{3}{5}, \text{ with } T = s\frac{3}{5}.$$

Thus, we get:

$$s = \frac{1}{3}.$$

Exercise 3

6 points

This exercise describes what is known as the *tragedy of the commons*. Consider a lake that can be freely accessed by a potentially infinite number of fishermen. The cost of sending a boat out on the lake is $r > 0$. When b boats are sent out onto the lake, $f(b) = \sqrt{b}$ fishes are caught in total. So, each boat catches $f(b)/b$ fishes. The unit price at which fishermen can sell fishes is $p > 0$, it is not affected by the level of the catch from the lake (i.e. we are reasoning in partial equilibrium). Fishermen's outside option is 0 if they do not fish.

1. Show that the equilibrium number of boats sent out on the lake if fishermen take decentralized decisions can be expressed a: 1

$$b^* = \left(\frac{p}{r}\right)^2$$

Since there is free entry in this industry and fishermen's outside option is 0, boats will be sent out on the lake until profit is positive. The equilibrium number of bots will thus be such that each boat's profit is null, i.e.:

$$p\frac{f(b)}{b} - r = 0 \Leftrightarrow \frac{f(b)}{b} = \frac{r}{p} \Leftrightarrow b^* = \left(\frac{p}{r}\right)^2.$$

2. Determine b^o, the number of boats that maximizes total social surplus. 1

Total social surplus can be expressed as:

$$\mathbb{W} = p \times f(b) - r \times b.$$

The first order condition with respect to b is:

$$p\frac{\partial f(b)}{\partial b} = r \Leftrightarrow p\frac{1}{2}b^{-1/2} = r \Leftrightarrow b^o = \frac{1}{4}\left(\frac{p}{r}\right)^2.$$

3. Compare b^o and b^*. Why don't the two values coincide? 2

It is clear that $b^o < b^*$. In other terms, to many fishes are caught when fishermen take decentralized decisions. The origin of this situation is that every new boat sent out on the lake yields a positive profit for the fisherman, but also reduced the quantity of fishes caught by other boats. This a negative externality caused by the fact that fishermen exploit a common resource.

4. What per-boat tax t would allow to restore efficiency? 2

We now assume that fishermen have to pay a tax t for each active boat. Following the same reasoning as in question 1, boats will be sent out on the lake until:

$$p\frac{f(b)}{b} - r - t = 0.$$

Thus, the number of boats will be:

$$b^t = \left(\frac{p}{r+t}\right)^2.$$

Since we want $b^t = b^o$, we just need to find t such that:

$$\left(\frac{p}{r+t}\right)^2 = \frac{1}{4}\left(\frac{p}{r}\right)^2 \Leftrightarrow t = r.$$