Public Economics

Final exam

Solutions

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The exam lasts 90 minutes. Documents are not allowed. The use of a calculator is allowed. Any other electronic devices are forbidden. You can answer either in French or in English.

> Question 1 is from *Intermediate Public Economics*, by J. Hindriks and G.D. Myles.

Question 1 — 3 points

Why is there more litter along highways than in people's yards?

> One candidate reason may be that litter along highways are cleaned by the local government authorities paid for from tax revenues whereas litter in people's yards are cleaned by individuals. As a consequence, individuals do not fully internalize the cleaning cost of litter along highways. Thus, they over-throw litter along highways with respects to the quantity they would throw in their yards. Here, the issue is that people are not aware of the ways the marginal cleaning cost translates into taxes.
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> Another candidate explanation may be simply that the cost suffered by individuals is much higher when they observe litter in their yards compared to litter along highways. This could be explained by the fact that people may spend less time on highways than in their yards. In relation to this point, highways can be viwed as an over-exploited common resource whereas people's yards are certainly considered as private resources.

Question 2 — 3 points

For which efficiency reason would we like to use lump-sum taxation? In other words, why do we say that not using lump-sum taxation results in second-best allocations? Also explain why lump-sum taxation is only rarely used.

The second welfare theorem tells us that any efficient allocation can be reached thanks to a decentralized equilibrium if starting endowments are set adequately. This is what lump-sum taxation of individuals achieves. However, lump-sum taxation is hardly feasible a large scale because it requires that the government (the one that redistributes) is able to have perfect information to design transfers between agents. In contrast, not using lump-sum taxation amounts to set taxes based on observable quantities such as income or consumption expenditure. The drawback of proportional or progressive taxation based on observable quantities is that it amounts to change relative prices in the economy, which results in second-best allocations (i.e. there is a deadweight loss with respect to the first-best efficient allocation).

Exercise 1 — 7 points

We consider an economy populated by two individuals—indexed by $i = 1, 2$—who have different preferences. Specifically, individual i's preferences over consumption c and labor l are given by:

$$u_i(c, l) = c - \frac{l^{1+\mu_i}}{1+\mu_i},$$

where $\mu_i > 0$. An individual with hourly wage w supplying labor l, earns $z = wl$ (pre-tax earnings) and consumes $c = z(1-\tau)$, where τ is the tax rate on labor income.

1. Show that the optimal labor supply by individual i is: 1

$$l_i^* = [w(1-\tau)]^{\frac{1}{\mu_i}}.$$

Individual i optimization program is:

$$\begin{array}{ll} \max_{c,l} & u_i(c,l), \\ \text{s.t.} & c \leq wl(1-\tau). \end{array}$$

The optimality condition is:

$$w(1-\tau) = l^{\mu_i}.$$

Thus, the optimal labor supply of individual i is:

$$l_i^* = [w(1-\tau)]^{\frac{1}{\mu_i}}$$

Let us now assume that the government is able to set a different tax rate τ_i for each individual i.

2. Determine optimal τ_1 and τ_2 that allow the government to maximize its total revenue. 2

$\mathbb{R}$, the government's total revenue can be written as:

$$\mathbb{R} = wl_1^*\tau_1 + wl_2^*\tau_2.$$

Choosing τ_1 and τ_2 to maximize $\mathbb{R}$ provide us with two distinct first order conditions:

$$(1-\tau_1)^{\frac{1}{\mu_1}} - \tau_1 \frac{1}{\mu}(1-\tau_1)^{\frac{1}{\mu_1}-1} = 0,$$

and,

$$(1-\tau_2)^{\frac{1}{\mu_2}} - \tau_2 \frac{1}{\mu}(1-\tau_2)^{\frac{1}{\mu_2}-1} = 0.$$

We get:

$$\tau_1 = \frac{1}{1+\frac{1}{\mu_1}}, \text{ and } \tau_2 = \frac{1}{1+\frac{1}{\mu_2}}$$

3. Interpret $\frac{1}{\mu_i}$. Comment on the relative values of τ_1 and τ_2 depending on μ_1 and μ_2. What is the general taxation principle illustrated here? 3

$\frac{1}{\mu_i}$ represents the elasticity of income with respect to tax rate $1-\tau$. It measures how individual i changes its labor supplies (and, consequently, its income) when the tax rate changes. Thus, the above formula implies that the tax rate τ_i will be larger for individuals that react more to tax changes. Here, $\mu_1 > \mu_2$ implies that $\tau_1 > \tau_2$. This illustrates the basic principle of optimal taxation that requires to tax more what is less elastic.

4. Further discuss depending on fairness considerations and on potential differences in wages across individuals. 1

Let us assume that individuals differ depending on their labor supply elasticity and on their wages. The optimal tax rates that we derived above imply that people whose labor supply is less elastic should be taxed more. Wage do not enter formulas. Now, if we assume that the poorest have both lower labor supply elasticity and wages, this means that they will be taxed much heavier than the richest. This may be considered as unfair.

Exercise 2 7 points

Let us consider and economy populated by three individuals—indexed by $i = 1, 2, 3$— who derive utility from the consumption of a public and a private good. All have similar preferences that are represented by the following utility function:

$$u_i = x_i G,$$

where x_i is the quantity of private good consumed by individual i and G is the total available quantity of public good. All individuals have the same income, such that $w_1 = w_2 = w_3 = 1$. The unit-price of the private good is 1. The cost of producing one unit of the public good is also 1.

1. Determine the equilibrium allocation if the public good is financed thanks to individuals' voluntary contributions g_1, g_2, and g_3. 1

Given prices and wages, we can use the budget constraint to write $x_i = 1 - g_i$. Each individual i decides on g_i such as to maximize its utility $(1-g_i)(g_1+g_2+g_3)$, taking others' contributions as given. For $i=1$, we get:

$$\frac{\partial u_1}{\partial g_1} = 1 - 2g_1 - g_2 - g_3 \Leftrightarrow g_1 = \frac{1-g_2-g_3}{2}.$$

Since all individuals are identical, we know that the equilibrium will be such that $g_1 = g_2 = g_3 \equiv g$. Thus, we get:

$$g = \frac{1}{4} \text{ and } G = 3g = \frac{3}{4}.$$

2. Show that the efficient allocation is such that $G = \frac{3}{2}$. 2

Thanks to Samuelson rule, we know that the efficient allocation is such that the sum of marginal rates of substitution is equal to the marginal rate of technical substitution. Let us use again the fact that individuals are identical. We get:

$$3 \times \frac{1-g}{3g} = 1 \Leftrightarrow g = \frac{1}{2} \text{ and } G = \frac{3}{2}.$$

3. Quickly check that the efficient allocation is Pareto-superior with respect to the one obtained thanks to voluntary contributions. Explain why they differ. 1

We say that an allocation is Pareto-superior to another one if at least one individual is better off in the former than in the latter and if nobody is worst off.

Here, all individuals get $u = \frac{1}{4}\frac{3}{4} = \frac{3}{16}$ when the public good is financed thanks to individuals' voluntary contributions. In contrast they all get $u = \frac{1}{2}\frac{3}{2} = \frac{12}{16}$ at the efficient allocation. Thus, the efficient allocation is Pareto-superior with respect to the one obtained thanks to voluntary contributions.

The reason of this difference is that when they take decentralized decisions, individuals do not take into account mutual positive externalities induced by the provision of the public good. As a result, they under-provide it.

Assume that the government is able to exclude individuals from the consumption of the public good. This implies that it is now possible to let each individual pay a unit price p to gain access to the total available quantity of the public good.

4. Determine p that allows to reach the efficient allocation. 2

We are looking for p such that each individual demands $G = \frac{3}{2}$ when maximizing the following utility function:

$$u_i = (1 - pG)G.$$

> The first order condition that determines the demand for G is:
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> $$1 - 2pG = 0 \Leftrightarrow G = \frac{1}{2p}.$$
>
> So, setting $p = \frac{1}{3}$ would do the job. Each individual will spend $pG = \frac{1}{2}$ in public good consumption, such that the total revenue for the producer will be $\frac{3}{2}$, what will allow to produce $G = \frac{3}{2}$ and to achieve the efficient allocation.

This equilibrium situation is known as Lindhal pricing. It amounts to let consumers pay an individualized price to access the public good.

5. All consumers are identical so far. What kind of issue would the government face if they were not? 1

> Setting personalized prices necessitates (i) to be able to distinguish between individuals and (ii) to be able to observe their willingness to pay for the public good (their demand for it). Both conditions are linked and the government may have a hard time pushing individuals to truly express their own preferences for the public good.