Public Economics

Optional intermediary exam

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The exam lasts 90 minutes. Documents are not allowed. The use of a calculator or of any other electronic devices is not allowed. You can answer either in French or in English.

> Exercises 1 and 3 are inspired from *Intermediate Public Economics*, by J. Hindriks and G.D. Myles.

Exercise 1 — 4 points

Consider an economy made of a government and a representative consumer who has preferences over consumption and leisure represented by the following utility function:

$$U = x(1 - l),$$

where x denotes consumption and l is labor time. The price of the consumption good is normalized to 1. When working, the consumer gets hourly wage w, considered as exogenous. Her labor income is thus wl. The government has two solutions to raise some revenue: either to set a lump-sum tax T, or to set a linear tax on labor income at rate t.

1. Determine l_T, the individual's labor supply under the lump-sum tax. 1

> The representative consumer maximizes its utility subject to the following budget constraint:
>
> $$x \leq wl - T.$$
>
> The first order condition with respect to l is:
>
> $$\frac{\partial U}{\partial l} = 0 \Leftrightarrow w - 2wl + T = 0,$$
>
> which yields:
>
> $$l_T = \frac{1}{2} + \frac{T}{2w}.$$

2. Determine l_t, the individual's labor supply under the linear tax on labor income. 1

The representative consumer maximizes its utility subject to the following budget constraint:

$$x \leq wl(1-t).$$

The first order condition with respect to l is:

$$\frac{\partial U}{\partial l} = 0 \Leftrightarrow (1-2l)(1-t)w = 0,$$

which yields:

$$l_t = \frac{1}{2}.$$

3. Determine R, the revenue raised by the labor income tax. Which of both solutions leads to the higher labor supply: the labor income tax at rate t or the lump-sum tax T that raises the same revenue as the labor income tax? Which solution should be chosen by the government? 2

Under the labor income tax, tax revenue can be expressed as:

$$R = l_t wt = \frac{1}{2}wt.$$

If the lump-sum tax raises the same revenue, then:

$$T = \frac{1}{2}wt.$$

Under such a lump-sum tax, the labor supply is:

$$l_T = \frac{1}{2} + \frac{\frac{1}{2}wt}{2w} = \frac{1}{2} + \frac{t}{4}.$$

For the same total tax revenue, labor supply is thus higher under the lump-sum tax than under labor income tax. This result comes from the fact that the use of the labor income tax distort the relative prices (here, the relative prices of leisure and consumption). In contrast, this is not the case with the lump-sum tax. Accordingly, the government should chose to use the lump-sum tax T rather than the labor income tax t.

Exercise 2 8 points

Consider an economy with N identical consumers indexed by $i = 1, \ldots, N$. Each of them has the same utility function:

$$U^i = \log(x^i) + \log(G),$$

where x^i is the consumption of a private good by individual i, and G is a pure public good. Each consumer is endowed with income 1. Let 1 be the unit price of the private good, such that each consumer's budget constraint can be written as $x^i + g^i \leq 1$, where g^i is individuals i's contribution to the public good. Total available quantity of the public good is the sum of individual contributions, i.e.:

$$G = \sum_{i=1}^{N} g^i.$$

It can be shown that the equilibrium public good provision when individuals take decentralized decisions is:

$$G^d = Ng = \frac{N}{N+1},$$

with $g^i = g, \forall i$ since individuals are identical.

1. Let us define G^o as the optimum public good provision when a social planner chooses the level of public good such has each individual contributes equally and the following social welfare function is maximized: 2

$$\mathbb{W} = \sum_{i=1}^{N} U^i.$$

Use Samuelson's rule to show that:

$$G^o = \frac{N}{2}.$$

According to Samuelson's rule, G^o must be such that the sum of individual marginal rates of substitution between the public and the public good is equal to the marginal rate of transformation between the two goods. Here, the technology is such that the latter is equal to 1. Thus, Samuelson's rule can be written as:

$$\sum_{i=1}^{N} \frac{\partial U^i/\partial g}{\partial U^i/\partial x} = 1 \Leftrightarrow \sum_{i=1}^{N} \frac{x}{G} = 1.$$

Since $x = 1 - g$, we get:

$$\sum_{i=1}^{N} \frac{1-g}{G} = 1 \Leftrightarrow \frac{1}{G}\sum_{i=1}^{N} 1 - g = 1 \Leftrightarrow \frac{1}{G}(N - G) = 1,$$

which yields:

$$G^o = \frac{N}{2}.$$

2. How could you justify public provision of the public good in this economy? 1

Here, we see that $G^d < G^o$ as long as $N > 1$. In other terms, there is under-provision of the public good. This under-provision occurs because individuals do not internalize the externality that their individual production of the public good creates on others. This externality is a failure of the second welfare theorem and calls for public intervention in the provision of the public good because all individuals would be better-off with more public good.

A government suddenly appears in this economy. It is endowed with the capacity to raise an identical lump-sum tax t on each individual and uses total taxes' revenue $T = Nt$ to produce some public good in quantity $\overline{G}$ using the following technology:

$$\overline{G} = \alpha \sum_{i=1}^{N} t = \alpha T,$$

with $\alpha > 0$. Accordingly, each individual's budget constraint is now $x^i + g^i + t \leq 1$. The total available quantity of the public good is now the sum of individual contributions and the publicly provided quantity, i.e.:

$$G = \sum_{i=1}^{N} g^i + \overline{G}.$$

3. Calculate $G^{d'}$, the equilibrium public good provision by private individuals only, when individuals take decentralized decisions under this new setting, i.e. individuals pay the tax t and consider $\overline{G}$ as given. Comment. 3

> Each individual chooses its contribution to the public good such as to maximize its own utility:
>
> $$\max_{g_i} U^i = \log(1 - g^i - t) + \log\left(g^i + \sum_{j=1, j\neq i}^{N} g^j + \overline{G}\right).$$
>
> The first order condition is:
>
> $$\frac{\partial U^i}{\partial g} = 0 \Leftrightarrow \frac{1}{1 - g_i - t} = \frac{1}{g_i + \sum_{j=1, j\neq i}^{N} g^j + \overline{G}}.$$
>
> Since all individuals are identical, $\forall i, g_i = g$, and since $\overline{G} = \alpha N t$, we get:
>
> $$g = \frac{1}{N+1} - \frac{\alpha N + 1}{N+1} t,$$
>
> which implies:
>
> $$G^{d'} = Ng = \frac{N}{N+1} - \frac{N}{N+1}(\alpha N + 1)t = \frac{N}{N+1}\left(1 - t\alpha N - t\right).$$
>
> Since $(1 - t\alpha N - t) < 1$, we see that $G^{d'} < G^d$. This is because of crowding out of private provision of the public good by the public provision of the public good : the latter decreases the former.

4. Calculate G^g, the total equilibrium public good provision, i.e. the sum of individual contributions, $G^{d'}$, and the publicly provided quantity, $\overline{G}$. 1

> G^g can be expressed as:
>
> $$G^g = G^{d'} + \alpha T = G^{d'} + \alpha N t,$$

which can be rewritten as:

$$G^g = \frac{N}{N+1}(1 - t\alpha N - t) + \alpha N t \Leftrightarrow G^g = \frac{N}{N+1}(1 - t\alpha N - t + \alpha(N+1)t)\,.$$

5. Discuss whether the government should engage in the provision of the public good depending on the value of α. 1

From the above expression, we can show easily that $G^g > G^d$ if and only if:

$$1 - t\alpha N - t + \alpha(N+1)t \Leftrightarrow \alpha > 1.$$

Thus, the government should engage in some provision of the public good if $\alpha > 1$. Since α represents the production technology of the government, this means that public provision of the public good is justified as long as the government's technology is more efficient than the private technology. In that case, we could determine an optimal level of the tax $t^* > 0$ such that aggregate social welfare is maximized. In contrast, if the government is less efficient than individuals in providing the public good, i.e. if $\alpha < 1$, then the government should not engage in the provision of the public good and set $t = 0$.

Exercise 3 8 points

Consider a situation where N individuals have to take a collective decision over multiple possible alternative choices. Each individual has its own preference ordering over alternatives.

1. Briefly define what is a Condorcet winner and present the five axioms used in Arrow's impossibility theorem. 2

The Condorcet winner is the alternative that defeats all other alternatives in pairwise majority voting.

The five axioms used by Arrow in his impossibility theorem are: (i) non-dictatorship (the social ordering should not be imposed by any individual), (ii) collective rationality (the social ranking must be complete, transitive, and reflexive), (iii) unrestricted domain (the decision rule must apply to all type of individual preferences), (iv) weak Pareto principle (if an alternative if preferred to another by all individuals, then the former should be socially preferred to the latter), and (v) independence of irrelevant alternatives (the social ordering of two alternative must depend only on individuals' preferences toward these two alternatives).

Let us consider the following preferences of five voters (indexed by $i = 1, \ldots, 5$) over five alternative choices (labeled $a, \ldots, e$):

	1	2	3	4	5
Most preferred alternative	a	b	c	d	e
	b	c	b	c	d
	e	a	e	a	c
	d	d	d	e	a
Least preferred alternative	c	e	a	b	b

The single transferable vote system, also known as the Hare procedure, has been developed in the 19th century by Thomas Hare. It was first used in 1896 to elect representatives at the Tasmanian House of Assembly. Nowadays, this system is used in some localities or regions to elect officials in a handful of countries. The procedure work as follows. Select the Condorcet winner if it exists. If not, the least desirable alternative(s) – defined as the alternative(s) that is (are) ranked first by the fewest number of voters – are successively deleted until a Condorcet winner is found among the remaining alternatives.

2. Applying the Hare procedure on the above profile of preferences, what is the social choice that emerges? Explain in detail the successive steps. 2

> Let us first look for a Condorcet winner. A quick investigation reveals that none of the alternatives is a Condorcet winner: in pairwise majority voting, a looses against c, c looses against d, d looses against e, e looses against b, and b looses against a.
>
> Thus, we have to delete alternatives that are ranked first by the fewest number of voters. Here, all alternatives are ranked first by only one voter. So, we have to delete all alternative.
>
> The Hare procedure does not work with this profile of preferences and no social choice emerges.

The Coombs procedure has been developed in the 20th century by Clyde Coombs. It operates as the Hare procedure, but instead of deleting alternatives with the fewest first places, it deletes alternatives with the most last places.

3. Applying the Coombs procedure on the above profile of preferences, what is the social choice that emerges? Explain in detail the successive steps. 2

> As shown above, there is no Condorcet winner in the first place. We have to delete alternatives with the most last places. Here, a is ranked last by one voter, b by two, c by one, d by zero, and e by one. Thus, we delete alternative b and the profile of preferences becomes:

	1	2	3	4	5
Most preferred alternative	a	c	c	d	e
	e	a	e	c	d
	d	d	d	a	c
Least preferred alternative	c	e	a	e	a

There is still no Condorcet winner: in pairwise majority voting, c wins over a, a wins over e, e wins over d, and d wins over c. Let us continue to delete alternatives with the most last places. Now, a and e are both ranked last by two voters. Thus, we delete alternatives a and e. The profile of preferences becomes:

	1	2	3	4	5
Most preferred alternative	d	c	c	d	d
Least preferred alternative	c	d	d	c	c

Now, alternative d wins the majority voting against c and emerges as social choice.

Let us consider the following preferences of four voters (indexed by $i = 1, \ldots, 4$) over three alternative choices (labeled a, b, c):

	1	2	3	4
Most preferred alternative	a	a	b	c
	b	b	c	b
Least preferred alternative	c	c	a	a

4. Applying the Hare procedure on the above preference profile will select alternative a as social choice. Use this preference profile to show that the Hare procedure violates Arrow's axiom of independence of irrelevant alternatives (this is also the case for the Coombs procedure, but you are not asked to show it). 2

Let us assume that voter 4's preferences toward b and c change. The degenerated profile of preferences is:

	1	2	3	4
Most preferred alternative	a	a	b	b
	b	b	c	c
Least preferred alternative	c	c	a	a

Note that the preferences of voter 4 toward a relatively to b and c did not change. Applying the Hare procedure, we will need to delete alternative c which is ranked first by nobody. We would end-up with the following profile:

	1	2	3	4
Most preferred alternative	a	a	b	b
Least preferred alternative	b	b	a	a

This profile of preferences fails to produce a Condorcet winner. The procedure is inconclusive. Thus, the Hare procedure is not independent of irrelevant alternatives: changing the relative preferences of one voter toward b and c turns a from the selected social choice to the impossibility to conclude.