Public Economics

Optional intermediary exam

Marc Sangnier - marc.sangnier@univ-amu.fr

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The exam lasts 90 minutes. Documents are not allowed. The use of a calculator or of any other electronic devices is not allowed. You can answer either in French or in English.

Exercise 1

4 points

Consider an economy made of a government and a representative consumer who has preferences over consumption and leisure represented by the following utility function:

$$U = x(1 - l),$$

where x denotes consumption and l is labor time. The price of the consumption good is normalized to 1. When working, the consumer gets hourly wage w, considered as exogenous. Her labor income is thus wl. The government has two solutions to raise some revenue: either to set a lump-sum tax T, or to set a linear tax on labor income at rate t.

1. Determine l_T, the individual's labor supply under the lump-sum tax. 1
2. Determine l_t, the individual's labor supply under the linear tax on labor income. 1
3. Determine R, the revenue raised by the labor income tax. Which of both solutions leads to the higher labor supply: the labor income tax at rate t or the lump-sum tax T that raises the same revenue as the labor income tax? Which solution should be chosen by the government? 2

Exercise 2

8 points

Consider an economy with N identical consumers indexed by $i = 1, \ldots, N$. Each of them has the same utility function:

$$U^i = \log(x^i) + \log(G),$$

where x^i is the consumption of a private good by individual i, and G is a pure public good. Each consumer is endowed with income 1. Let 1 be the unit price of the private good, such that each consumer's budget constraint can be written as $x^i + g^i \leq 1$, where g^i is individuals i's contribution to the public good. Total available quantity of the

public good is the sum of individual contributions, i.e.:

$$G = \sum_{i=1}^{N} g^i.$$

It can be shown that the equilibrium public good provision when individuals take decentralized decisions is:

$$G^d = Ng = \frac{N}{N+1},$$

with $g^i = g, \forall i$ since individuals are identical.

1. Let us define G^o as the optimum public good provision when a social planner chooses the level of public good such has each individual contributes equally and the following social welfare function is maximized: 2

$$\mathbb{W} = \sum_{i=1}^{N} U^i.$$

Use Samuelson's rule to show that:

$$G^o = \frac{N}{2}.$$

2. How could you justify public provision of the public good in this economy? 1

A government suddenly appears in this economy. It is endowed with the capacity to raise an identical lump-sum tax t on each individual and uses total taxes' revenue $T = Nt$ to produce some public good in quantity $\overline{G}$ using the following technology:

$$\overline{G} = \alpha \sum_{i=1}^{N} t = \alpha T,$$

with $\alpha > 0$. Accordingly, each individual's budget constraint is now $x^i + g^i + t \leq 1$. The total available quantity of the public good is now the sum of individual contributions and the publicly provided quantity, i.e.:

$$G = \sum_{i=1}^{N} g^i + \overline{G}.$$

3. Calculate $G^{d'}$, the equilibrium public good provision by private individuals only, when individuals take decentralized decisions under this new setting, i.e. individuals pay the tax t and consider $\overline{G}$ as given. Comment. 3

4. Calculate G^g, the total equilibrium public good provision, i.e. the sum of individual contributions, $G^{d'}$, and the publicly provided quantity, $\overline{G}$. 1

5. Discuss whether the government should engage in the provision of the public good depending on the value of α. 1

Exercise 3

8 points

Consider a situation where N individuals have to take a collective decision over multiple possible alternative choices. Each individual has its own preference ordering over alternatives.

1. Briefly define what is a Condorcet winner and present the five axioms used in Arrow's impossibility theorem. 2

Let us consider the following preferences of five voters (indexed by $i = 1, \ldots, 5$) over five alternative choices (labeled $a, \ldots, e$):

	1	2	3	4	5
Most preferred alternative	a	b	c	d	e
	b	c	b	c	d
	e	a	e	a	c
	d	d	d	e	a
Least preferred alternative	c	e	a	b	b

The single transferable vote system, also known as the Hare procedure, has been developed in the 19$^{\text{th}}$ century by Thomas Hare. It was first used in 1896 to elect representatives at the Tasmanian House of Assembly. Nowadays, this system is used in some localities or regions to elect officials in a handful of countries. The procedure work as follows. Select the Condorcet winner if it exists. If not, the least desirable alternative(s) – defined as the alternative(s) that is (are) ranked first by the fewest number of voters – are successively deleted until a Condorcet winner is found among the remaining alternatives.

2. Applying the Hare procedure on the above profile of preferences, what is the social choice that emerges? Explain in detail the successive steps. 2

The Coombs procedure has been developed in the 20$^{\text{th}}$ century by Clyde Coombs. It operates as the Hare procedure, but instead of deleting alternatives with the fewest first places, it deletes alternatives with the most last places.

3. Applying the Coombs procedure on the above profile of preferences, what is the social choice that emerges? Explain in detail the successive steps. 2

Let us consider the following preferences of four voters (indexed by $i = 1, \ldots, 4$) over three alternative choices (labeled a, b, c):

	1	2	3	4
Most preferred alternative	a	a	b	c
	b	b	c	b
Least preferred alternative	c	c	a	a

4. Applying the Hare procedure on the above preference profile will select alternative a as social choice. Use this preference profile to show that the Hare procedure violates Arrow's axiom of independence of irrelevant alternatives (this is also the case for the Coombs procedure, but you are not asked to show it). 2