Public Economics

Final exam

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The exam lasts 90 minutes. Documents are not allowed. The use of a calculator is allowed. Any other electronic devices are forbidden. You can answer either in French or in English.

> Questions 1 and 2 are inspired from *Intermediate Public Economics*, by J. Hindriks and G.D. Myles. Exercise 1 is inspired from a problem set by J. Poterba, I. Werning, and D. Struyven. Exercise 2 is inspired from a a problem set by T. Piketty.

Question 1

4 points

Comment on the following statement: "Since pollution is bad, it would be socially optimal to prohibit the use of any production process that creates pollution."

> This statement hinges on the following reasoning: One should avoid any activity that negatively affects the utility function. Yet, this in not consistent with economic reasoning that suggests to weight costs and benefits of any decision or action. What the above reasoning misses is that production processes that create pollution also create goods are services that enter positively in individuals' utility. Hence, setting the production of polluting goods and services to zero is likely to be sub-optimal. The only case in which this would be optimal is the case where the social marginal cost (i.e. the marginal cost of production factors plus the marginal damage from pollution) of the very first unit already exceeds the social marginal benefit (i.e. the marginal utility) derived from the consumption of this first unit.

Question 2

4 points

Assume that, thanks to high-altitude winds, all our polluting emissions are blown into neighboring countries. Can our national economy be efficient? Discuss depending on whether polluting emissions have world-wide environmental consequences (e.g. unpleasant climatic change) or only local ones.

> If all our polluting emissions are blown into neighboring counties, this means that the total social cost of our production falls into these countries. As a consequence, private and social cost coincide within our country, what leaves the possibility that social costs and benefits are equal in our country. Our economy is thus efficient.

Yet, this only holds if polluting emissions have no world-wide environmental consequences. If they do, then polluting emissions will still increase our social cost of production despite the fact that they are blown into neighboring countries. This will create a gap between social and private cost. Our economy would thus be inefficient.

Exercise 1

6 points

Consider an individual with preferences over consumption in two periods given by:

$$V(C_1, C_2) = \log(C_1) + \frac{1}{1+\delta}\log(C_2),$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively, and δ is the rate of time preference. This individual receives labor income Y_1 in period 1, and Y_2 in period 2. Labor income is taxed at rate τ_1 in period 1, and at rate τ_2 in period 2. The individual can borrow or lend at rate r. She also have access to a tax avoidance technology that allows her to shift labor income from period 1 to period 2. If the individual chooses to shift $A \in [0, Y_1]$ euro from period 1 to period 2, her taxable income in the first period will be $Y_1 - A$ and that in period 2 will be $Y_2 + A$. Shifting A euro costs $\beta(A)$ euro, with $\beta'(A) > 0$, $\beta''(A) > 0$, $\beta(0) = 0$, and $\beta'(0) = 0$. This cost must be paid in period 1.

1. Remember that, in the absence of both taxes and tax avoidance technology, the individual's intertemporal budget constraint would be:

$$C_1 + \frac{1}{1+r}C_2 \leq Y_1 + \frac{1}{1+r}Y_2.$$

Determine the individual's intertemporal budget constraint with taxes and tax avoidance technology. 1

The intertemporal budget constraint is:

$$C_1 + \frac{1}{1+r}C_2 + \beta(A) \leq (1-\tau_1)(Y_1 - A) + \frac{1-\tau_2}{1+r}(Y_2 + A).$$

2. Write down the individual's maximization program. Explain why the optimal level of shifting chosen by the individual will not depend on the utility function. 1

Each individual optimally chooses consumed quantities and how much income to shift from the first to the second period. Thus, the individual's maximization program can be written as:

$$\begin{array}{ll} \max_{C_1, C_2, A} & V(C_1, C_2), \\ \text{s.t.} & C_1 + \frac{1}{1+r}C_2 + \beta(A) \leq (1-\tau_1)(Y_1 - A) + \frac{1-\tau_2}{1+r}(Y_2 + A). \end{array}$$

The optimal level of shifting chosen by the individual will not depend on the utility function because it does not affect directly utility. It will be chosen such as to maximize the presented value of wealth (i.e. the right-hand term of the intertemporal budget constraint). This becomes clear when looking at the maximization program: the first order condition with respect to A will not have any term from the utility function.

3. The first order optimality condition that defines A^*, the optimal level of income shifting, can be written as:

$$\beta'(A^*) = \frac{1-\tau_2}{1+r} - (1-\tau_1).$$

Comment. 1

The left-hand term of this condition represent the marginal cost of shifting one more euro from one period to the other. The right-hand term represent the marginal benefit from shifting income. This can decomposed in two parts as shifting one more euro will (i) increase the budget available in period 2, and (ii) reduce the feasible consumption in period 1. This optimality condition simply reflects the fact that A^*, the optimal level of income shifting, must be such that marginal benefit of shifting equals marginal cost of shifting.

4. In what case will there be no tax avoidance? Was this to be expected? 1

There will be no tax avoidance if the solution of the above first order condition is such that $A^* = 0$. We know that this occurs if and only if $\beta'(0) = 0$. Or equivalently, if:

$$\frac{1-\tau_2}{1+r} - (1-\tau_1) \leq 0.$$

This expression can be rewritten as:

$$\frac{1-\tau_2}{1+r} \leq (1-\tau_1).$$

The left-hand term of this expression represents the present value of a one euro income in period 2, net of taxation. The right-hand term represents the value of one euro in period 1, net of taxation. In other words, there will be not tax avoidance, i.e. A^* will be equal to 0, if tax rates, τ_1 and τ_2, are such that income is less taxed in the first period than in the second period. Note that, using outrageous mathematical approximations, the above condition can be rewritten as:

$$\tau_1 \leq \tau_2 + r.$$

Alternatively, it can be rewritten as:

$$\tau_1 \leq \tau_2,$$

if one sets $r = 0$.

5. Consider the case in which $\beta(A) = \gamma A^2$, with $\gamma > 0$. Further assume that $r = 0$, and note that government' total tax revenues are equal to:

$$R = \tau_1(Y_1 - A^*) + \tau_2(Y_2 + A^*).$$

What are the implications on tax revenues of raising τ_1 or τ_2? Discuss the mechanisms at play in both cases. 2

If $\beta(A) = \gamma A^2$ and r=0, the optimality condition can be written as:

$$A^* = \frac{\tau_1 - \tau_2}{2\gamma}.$$

Thus, taking into account the individual optimal choice, government' total tax revenues can be written as:

$$R = \tau_1 \left[Y_1 - \frac{\tau_1 - \tau_2}{2\gamma} \right] + \tau_2 \left[Y_2 + \frac{\tau_1 - \tau_2}{2\gamma} \right].$$

From now on, let us only focus on the case where $\tau_1 > \tau_2 + r$ since we know that there is no tax avoidance in the other case. The derivative of R with respect to τ_1 is:

$$\frac{\partial R}{\partial \tau_1} = Y_1 - 2A^* = Y_1 - \frac{\tau_1 - \tau_2}{\gamma},$$

which has an ambiguous sign. This ambiguity arises because raising τ_1 has two opposite effects: (i) it mechanically raises first period tax revenues, and (ii) it give more incentives to shift income to the next period, what reduces first period tax revenues but is compensated by second period tax revenues. Similarly, the derivative of R with respect to τ_2 is:

$$\frac{\partial R}{\partial \tau_2} = Y_2 + 2A^* = Y_2 + \frac{\tau_1 - \tau_2}{\gamma},$$

which is unambiguously positive. This positive effect is a combination of two positive effects as raising τ_2 (i) mechanically raises second period tax revenues, and (ii) gives less incentive to shift income from the first to the second period, what increases first period tax revenues.

Exercise 2 6 points

We consider an economy made of individuals who receive the same hourly wage w but have different preferences. Specifically, individual i's preferences over consumption c and labor l are given by:

$$u_i(c, l) = c - \frac{l^{1+\mu_i}}{1 + \mu_i},$$

where $\mu_i > 0$. An individual with wage w supplying labor l, earns $z = wl$ (pre-tax earnings) and consumes $c = z(1 - \tau)$, where τ is the tax rate on labor income.

1. Compute the optimal labor supply that individual i makes. 1

Individual i optimization program is:

$$\begin{array}{ll} \max_{c,l} & u_i(c, l), \\ \text{s.t.} & c \leq wl(1 - \tau). \end{array}$$

The optimality condition is:

$$w(1 - \tau) = l^{\mu_i}.$$

Thus, the optimal labor supply of individual i is:

$$l_i^* = [w(1-\tau)]^{\frac{1}{\mu_i}}$$

Assume that the government is able to set a different tax rate τ_i for each individual i.

2. Show that total tax revenue will be maximized if the government set tax rates such as:

$$\forall i,\ \tau_i = \frac{1}{1+\frac{1}{\mu_i}}.$$

1

Total tax revenue R is the sum of taxes levies on all individuals, i.e.:

$$R = \sum_i \tau_i w l_i^*,$$

where:

$$l_i^* = [w(1-\tau_i)]^{\frac{1}{\mu_i}}.$$

The government chooses all τ_is such as to maximize R. Since the expression of R is additively separable, we obtain a set of optimality conditions such as:

$$\forall i,\ \frac{\partial R}{\partial \tau_i} = 0.$$

The above expression can be rewritten as:

$$\forall i,\ (1-\tau_i)^{\frac{1}{\mu_i}} - \tau_i \frac{1}{\mu}(1-\tau_i)^{\frac{1}{\mu_i}-1} = 0,$$

which yields:

$$\forall i,\ \tau_i = \frac{1}{1+\frac{1}{\mu_i}}.$$

3. What does $\frac{1}{\mu_i}$ represent? Comment on the above formula. 2

$\frac{1}{\mu_i}$ represents the elasticity of income with respect to tax rate $1-\tau$. It measures how individual i changes its labor supplies (and, consequently, its income) when the tax rate changes. Thus, the above formula implies that the tax rate τ_i will be larger for individuals that react more to tax changes. This illustrates the basic principle of optimal taxation that requires to tax more what is less elastic.

For some technical reasons, the government is not able to set a different tax rate for each individual i. Accordingly, the government decides to set a common tax rate $\overline{\tau}$ such as:

$$\overline{\tau} = \frac{1}{1+\mathbb{E}\left(\frac{1}{\mu}\right)},$$

where $\mathbb{E}\left(\frac{1}{\mu}\right)$ is the average of $\frac{1}{\mu}$ over the whole population.

4. Comment on this solution. 2

This solution reflects an information problem: the government is not able to observe each individual labor supply elasticity. As a consequence, the government must rely on some common tax rate that will be—from the viewpoint of maximizing tax revenues—to high for some individuals and to low for other individuals. As a consequence, tax revenues will not be maximized. This is a second-best solution that meets technical constraints. Note that we cannot say anything about the optimality or non-optimality of this solution from the viewpoint of social welfare as we did not model the way taxes are used, nor how government's production enter the utility function. Taking the model at face value, social welfare would be maximized if the tax rate was set to 0.